Indonesian Journal of Combinatorics **3** (2) (2019), 109–115

INDONESIAN JOURNAL OF COMBINATORICS

On size multipartite Ramsey numbers for stars

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Abstract

Burger and Vuuren defined the size multipartite Ramsey number for a pair of complete, balanced, multipartite graphs $m_j(K_{a\times b}, K_{c\times d})$, for natural numbers a, b, c, d and j, where $a, c \geq 2$, in 2004. They have also determined the necessary and sufficient conditions for the existence of size multipartite Ramsey numbers $m_j(K_{a\times b}, K_{c\times d})$. Syafrizal *et. al.* generalized this definition by removing the completeness requirement. For simple graphs G and H, they defined the size multipartite Ramsey number $m_j(G, H)$ as the smallest natural number t such that any red-blue coloring on the edges of $K_{j\times t}$ contains a red G or a blue H as a subgraph. In this paper, we determine the necessary and sufficient conditions for the existence of multipartite Ramsey numbers $m_j(G, H)$, where both G and H are non complete graphs. Furthermore, we determine the exact values of the size multipartite Ramsey numbers $m_j(K_{1,m}, K_{1,n})$ for all integers $m, n \geq 1$ and $j = 2, 3$, where $K_{1,m}$ is a star of order $m + 1$. In addition, we also determine the lower bound of $m_3(kK_{1,m}, C_3)$, where $kK_{1,m}$ is a disjoint union of k copies of a star $K_{1,m}$ and C_3 is a cycle of order 3.

Keywords: cycle, existence, size multipartite Ramsey number, star.
Mathematics Subject Classification : 05C55
DOI: 10.19184/ijc.2019.3.2.4

1. Introduction

The classical Ramsey number $r(a, c)$ is the smallest natural number j such that any red-blue coloring of the edges of K_j, necessarily forces a red K_a or a blue K_c as subgraph. The size multipartite Ramsey number is one of generalizations of the classical Ramsey number. Burger and Vuuren [1] gave a definition of the size multipartite Ramsey numbers for a pair of complete, balanced, multipartite graphs, as follows. Let a, b, c, d and j, be natural numbers with $a, c \geq 2$, the

Received: 6 Jul 2019, Revised: 25 Nov 2019, Accepted: 5 Dec 2019.

size multipartite Ramsey number $m_j(K_{a\times b}, K_{c\times d})$ is the smallest natural number t such that any red-blue coloring of the edges of $K_{j\times t}$, necessarily forces a red $K_{a\times b}$ or a blue $K_{c\times d}$ as subgraph. They also determined $m_j(K_{2\times 2}, K_{3\times 1})$, for $j \geq 1$ and have established the following existence of size multipartite Ramsey numbers.

Theorem 1.1. *(The existence of size numbers)* [1]
The size multipartite Ramsey numbers $m_j(K_{a\times b}, K_{c\times d})$ exists for any $a, c \geq 2$ and $b, d \geq 1$ if and only if $j \geq r(a, c)$.

Syafrizal *et. al.* [10] generalized this definition by removing the completeness requirement. For simple graphs G and H, they defined the size multipartite Ramsey number $m_j(G, H)$ as the smallest natural number t such that any red-blue coloring on the edges of $K_{j\times t}$ contains a red G or a blue H as a subgraph. The size bipartite Ramsey numbers for stars versus paths $m_2(K_{1,m}, P_n)$, for $m, n \geq 2$ given by Hattingh and Henning [3]. In 2007, Syafrizal *et al.* [11] determined the size multipartite Ramsey numbers for stars versus P_3. Then, Surahmat *et al.* [9] gave the size tripartite Ramsey numbers for stars versus P_n, for $3 \leq n \leq 6$. Furthermore, we gave the size multipartite Ramsey numbers for stars versus cycles [5] and the size tripartite Ramsey numbers for a disjoint union of m copies of a star $K_{1,n}$ versus P_3 [6]. In 2017, Jayawardene *et al.* [4] and Effendi *et al.* [2] determined the size multipartite Ramsey numbers for stars versus paths. Then, we also gave the size multipartite Ramsey numbers for stars versus paths and cycles [7], that complete the previous results given by Syafrizal and Surahmat. Recently, we determined $m_j(mK_{1,n}, H)$, where $H = P_3$ or $K_{1,3}$ for $j \geq 3, m, n \geq 2$ [8].

In this paper, we determine the necessary and sufficient conditions for the existence of the size multipartite Ramsey numbers $m_j(G, H)$, where both G and H are non complete graphs. Furthermore, we determine the exact values of the size multipartite Ramsey numbers $m_j(K_{1,m}, K_{1,n})$ for all integers $m, n \geq 1$ and $j = 2, 3$. In addition, we also determine the lower bound of $m_3(kK_{1,m}, C_3)$.

We call some basic definitions that will be used in this paper, as follows. Let G be a finite and simple graph. Let vertex and edge sets of graph G are denoted by $V(G)$ and $E(G)$, respectively. Vertex colorings in which adjacent vertices are colored differently are *proper vertex colorings*. A graph G is *k-colorable* if there exists a proper vertex coloring of G from a set of k colors. A *matching* of a graph G is defined as a set of edges without a common vertex. A matching of maximum size in G is a *maximum matching* in G. The *maximum degree* of G is denoted by $\Delta(G)$, where $\Delta(G) = max\{d(v)|v \in V(G)\}$. The *minimum degree* of G is denoted by $\delta(G)$, where $\delta(G) = min\{d(v)|v \in V(G)\}$. A star $K_{1,n}$ is the graph on $n+1$ vertices with one vertex of degree n, called the *center* of this star, and n vertices of degree 1, called the *leaves*. A disjoint union of k copies of a star $K_{1,m}$, a cycle of order n, and a path of order n are denoted by $kK_{1,m}, C_n$, and P_n, respectively.

2. Results

For any non complete graphs G and H, we will determine the necessary and sufficient conditions for the existence of the size multipartite Ramsey numbers $m_j(G, H)$. In order to do so, we recall the definition of the *chromatic number* of a graph G, denoted by $\chi(G)$, which is the minimum positive integer k for which G is k-colorable.

Lemma 2.1. *In every proper vertex coloring of a simple graph G, the maximum number of the vertices in G with the same color is* $|V(G)| - \chi(G) + 1$.

Proof. Let c be a proper vertex coloring of G, with $\chi(G)$ color, that is $c : V(G) \to \{1, 2, ..., \chi(G)\}$. Let $C_i = \{v \in V(G)|c(v) = i\}$. Without lost generality, let $|C_1| \le |C_2| \le ... \le |C_{\chi(G)}|$. Since for $1 \le i \le \chi(G) - 1$, we have $|C_i| \ge 1$, then $|C_{\chi(G)}| \le |V(G)| - \chi(G) + 1$. □

Theorem 2.1. *Let G and H be two non complete graph. The multipartite Ramsey numbers* $m_j(G, H)$ *are finite if and only if* $j \ge$ *maks*$\{\chi(G), \chi(H)\}$.

Proof. Let $m_j(G, H) = t < \infty$, that is $K_{j\times t} \to (G, H)$. If $K_{j\times t} = F_1 \oplus F_2$, then ($F_1 \not\supseteq G \Rightarrow F_2 \supseteq H$) or ($F_2 \not\supseteq H \Rightarrow F_1 \supseteq G$). This implies that $j \ge \chi(H)$ and $j \ge \chi(G)\}$. Therefore, $j \ge$ maks$\{\chi(G), \chi(H)\}$.

Let $j \ge$ maks$\{\chi(G), \chi(H)\}$. We show that $m_j(G, H)$ is finite. We construct an positive integer t such that $K_{j\times t} \to (G, H)$. Let $p = |V(G)| - \chi(G) + 1, q = |V(H)| - \chi(H) + 1$ and $t = p + q$. Note that $V(K_{j\times t}) = V(K_{j\times p}) \cup V(K_{j\times q})$. Based on Lemma 2.1, p and q are the maximum number of the same colored vertices in G and H, respectively, so $K_{j\times p} \supseteq G$ and $K_{j\times q} \supseteq H$. Therefore, $K_{j\times t} \to (G, H)$. Then, $m_j(G, H) \le t$. Since graph G and H are finite graph, so $|V(G)|, |V(H)|, \chi(G)$ and $\chi(H)$ are finite. So, $m_j(G, H) \le t < \infty$. Then, $m_j(G, H)$ is finite. □

Theorem 2.2. *For positive integers* m *and* n, *we have* $m_2(K_{1,m}, K_{1,n}) = m + n - 1$.

Proof. We will show that $m_2(K_{1,m}, K_{1,n}) \ge m + n - 1$. We consider a red-blue coloring on the edges of graph $K_{2\times(m+n-2)} = F_R \oplus F_B$, such that F_R is a $(m-1)-$regular graph. By *Handshaking* Lemma, it is possible since the sum of the degrees of the vertices of F_R is even. Then, $F_R \not\supseteq K_{1,m}$. We have $d(v) = m + n - 2 - (m - 1) = n - 1$, for any v in F_B. Hence, $F_B \not\supseteq K_{1,n}$.

Now, we will show that $m_2(K_{1,m}, K_{1,n}) \le m+n-1$. We consider any red-blue coloring on the edges of graph $K_{2\times(m+n-1)} = G_R \oplus G_B$, such that $G_R \not\supseteq K_{1,m}$. This implies that $\Delta(G_R) \le m-1$. Therefore, $\delta(G_B) \ge m + n - 1 - (m - 1) = n$. Then, $G_B \supseteq K_{1,n}$. □

Theorem 2.3. *For positive integers* m *and* n, *we have*

$$m_3(K_{1,m}, K_{1,n}) = \begin{cases} \frac{m}{2}, & \textit{for } m \equiv 2 \textit{ mod } 4, n = 1, 2 \\ 2\lfloor\frac{m+1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil, & \textit{for } m \equiv 2 \textit{ mod } 4, n \equiv 3 \textit{ mod } 4, \\ 2\lfloor\frac{m-1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil, & \textit{for } m \equiv 4 \textit{ mod } 4, n \equiv 1 \textit{ mod } 4, \\ \frac{m-1}{2} + \lceil\frac{n}{2}\rceil, & \textit{for } m \equiv 1 \textit{ mod } 2, n \ge 1, \\ 2\lfloor\frac{m+1}{4}\rfloor + 2\lfloor\frac{n}{4}\rfloor + 1, & \textit{for } m \equiv 2 \textit{ mod } 4, n \not\equiv 3 \textit{ mod } 4, n \ge 4, \\ 2\lfloor\frac{m-1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil + 1, & \textit{for } m \equiv 4 \textit{ mod } 4, n \not\equiv 1 \textit{ mod } 4. \end{cases}$$

Proof. **Case 1**. $m_3(K_{1,m}, K_{1,n}) = \frac{m}{2}$, for $m \equiv 2$ mod 4, and $n = 1, 2$.

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For $n = 1$, we will use the property that $m_3(K_{1,m}, K_1) \leq m_3(K_{1,m}, K_{1,1})$. It is clear that $m_3(K_{1,m}, K_1) = \frac{m}{2}$. Therefore, $m_3(K_{1,m}, K_{1,1}) \geq \frac{m}{2}$. If $K_{3\times\frac{m}{2}}$ contains no a blue $K_{1,1}$, then $K_{3\times\frac{m}{2}}$ contains a red $K_{1,m}$, since $d(v) = m$, for any v in $K_{3\times\frac{m}{2}}$. Hence, $m_3(K_{1,m}, K_{1,1}) \leq \frac{m}{2}$.

For $m = n = 2$, it is clear that $m_3(K_{1,m}, K_{1,n}) \geq \frac{m}{2}$. For $m \equiv 6 \bmod 4$ and $n = 2$, we consider a red-blue coloring on the edges of graph $K_{3\times(\frac{m}{2}-1)}$, such that $K_{3\times(\frac{m}{2}-1)}$ contains a maximum blue matching graph. Since $\frac{m}{2} - 1$ is even, the blue graph is a $1-$regular graph. This implies that graph $K_{3\times(\frac{m}{2}-1)}$ contains red $(m-3)-$regular graph. So $K_{3\times(\frac{m}{2}-1)}$ contains no a red $K_{1,m}$. Then, $m_3(K_{1,m}, K_{1,2}) \geq \frac{m}{2}$. Furthermore, we consider any red-blue coloring on the edges of graph $K_{3\times\frac{m}{2}}$, such that graph $K_{3\times\frac{m}{2}}$ contains no a blue $K_{1,2}$. This implies that the maximum degree of blue graph is 1. Since $\frac{m}{2}$ is odd, then there is at least one vertex v, where $d(v) = 0$ in blue graph and $d(v) = m$ in red graph. Then, $K_{3\times\frac{m}{2}}$ contains a red $K_{1,m}$. Therefore, $m_3(K_{1,m}, K_{1,2}) \leq \frac{m}{2}$.

Case 2. For ($m \equiv 2 \bmod 4$ and $n \equiv 3 \bmod 4$), let $t = 2\lfloor\frac{m+1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil$ and for ($m \equiv 4 \bmod 4$ and $n \equiv 1 \bmod 4$), let $t = 2\lfloor\frac{m-1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil$.

We consider a red-blue coloring on the edges of graph $K_{3\times(t-1)} = F_R \oplus F_B$, such that $d(v_1) = m - 2$, for a vertex $v_1 \in V(F_R)$ and $d(v) = m - 1$, for any $v \in V(F_R) - \{v_1\}$. By *Handshaking* Lemma, it is possible since the sum of the degrees of the vertices of F_R is even. Then, $F_R \not\supseteq K_{1,m}$. We distinguish the following two cases, to show that $m_3(K_{1,m}, K_{1,n}) \geq t$.

Case a. For $m \equiv 2 \bmod 4$ and $n \equiv 3 \bmod 4$.

We have $d(v_1) = 2t - m = 4\lfloor\frac{m+1}{4}\rfloor + 4\lceil\frac{n}{4}\rceil - m = m - 2 + n + 1 - m = n - 1$, for $v_1 \in V(F_B)$ and $d(v) = 2t - m - 1 = 4\lfloor\frac{m+1}{4}\rfloor + 4\lceil\frac{n}{4}\rceil - m - 1 = m - 2 + n + 1 - m - 1 = n - 2$, for any $v \in V(F_B) - \{v_1\}$. Then, $F_B \not\supseteq K_{1,n}$.

Case b. For $m \equiv 4 \bmod 4$ and $n \equiv 1 \bmod 4$.

We have $d(v_1) = 2t - m = 4\lfloor\frac{m-1}{4}\rfloor + 4\lceil\frac{n}{4}\rceil - m = m - 4 + n + 3 - m = n - 1$, for $v_1 \in V(F_B)$ and $d(v) = 2t - m - 1 = 4\lfloor\frac{m+1}{4}\rfloor + 4\lceil\frac{n}{4}\rceil - m - 1 = m - 4 + n + 3 - m - 1 = n - 2$, for any $v \in V(F_B) - \{v_1\}$. Then, $F_B \not\supseteq K_{1,n}$.

Now, we consider any red-blue coloring on the edges of graph $K_{3\times t} = G_R \oplus G_B$, such that $G_R \not\supseteq K_{1,m}$. This implies that $\Delta(G_R) \leq m - 1$. We distinguish the following two cases, to show that $m_3(K_{1,m}, K_{1,n}) \leq t$.

Case a. For $m \equiv 2 \bmod 4$ and $n \equiv 3 \bmod 4$.

$\delta(G_B) \geq 2t - (m-1) = 2t - m + 1 = m - 1 + 2\lceil\frac{n}{2}\rceil - m + 1 = n + 1$, since n is odd. Then, $G_B \supseteq K_{1,n}$.

Case b. For $m \equiv 4 \bmod 4$ and $n \equiv 1 \bmod 4$.

$\delta(G_B) \geq 2t - (m-1) = 2t - m + 1 = 4\lfloor\frac{m-1}{4}\rfloor + 4\lceil\frac{n}{4}\rceil - m + 1 = m - 4 + n + 3 - m + 2 = n$. Therefore, $G_B \supseteq K_{1,n}$.

Case 3. For $m \equiv 1 \bmod 2$ and $n \geq 1$, let $t = \frac{m-1}{2} + \lceil\frac{n}{2}\rceil$, for $m \equiv 2 \bmod 4$ and $n \neq 3 \bmod 4$, let $t = 2\lfloor\frac{m+1}{4}\rfloor + 2\lfloor\frac{n}{4}\rfloor + 1$, and for $m \equiv 4 \bmod 4$ and $n \neq 1 \bmod 4$, let $t = 2\lfloor\frac{m-1}{4}\rfloor + 2\lceil\frac{n}{4}\rceil + 1$.

We consider a red-blue coloring on the edges of graph $K_{3\times(t-1)} = F_R \oplus F_B$, such that F_R is a $(m-1)-$regular graph. By *Handshaking* Lemma, it is possible since the sum of the degrees of the vertices of F_R is even. Then, $F_R \not\supseteq K_{1,m}$. We have $d(v) = 2(t-1) - (m-1)$. We distinguish the following three cases, to show that $m_3(K_{1,m}, K_{1,n}) \geq t$.

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Case a. For $m \equiv 1 \bmod 2$ dan $n \geq 1$.

$d(v) = 2t - m - 1 = m - 1 + 2\lceil \frac{n}{2} \rceil - m - 1 = 2\lceil \frac{n}{2} \rceil - 2 < n$, for any v in F_B. Then, $F_B \not\supseteq K_{1,n}$.

Case b. For $m \equiv 2 \bmod 4$ and $n \neq 3 \bmod 4$.

$d(v) = 2t - m - 1 = 4\lfloor \frac{m+1}{4} \rfloor + 4\lfloor \frac{n}{4} \rfloor + 2 - m - 1 = m - 2 + 4\lfloor \frac{n}{4} \rfloor - m + 1 = 4\lfloor \frac{n}{4} \rfloor - 1 \leq n - 1$, for any v in F_B. Then, $F_B \not\supseteq K_{1,n}$.

Case c. For $m \equiv 4 \bmod 4$ and $n \neq 1 \bmod 4$.

$d(v) = 2t - m - 1 = 4\lfloor \frac{m-1}{4} \rfloor + 4\lceil \frac{n}{4} \rceil + 2 - m - 1 = m - 4 + 4\lceil \frac{n}{4} \rceil - m + 1 = 4\lceil \frac{n}{4} \rceil - 3 < n$, for any v in F_B. Then, $F_B \not\supseteq K_{1,n}$.

Now, we consider any red-blue coloring on the edges of graph $K_{3\times t} = G_R \oplus G_B$, such that $G_R \not\supseteq K_{1,m}$. This implies that $\Delta(G_R) \leq m - 1$. We distinguish the following three cases, to show that $m_3(K_{1,m}, K_{1,n}) \leq t$.

Case a. For $m \equiv 1 \bmod 2$ dan $n \geq 1$.

$\delta(G_B) \geq 2t - (m-1) = 2t - m + 1 = m - 1 + 2\lceil \frac{n}{2} \rceil - m + 1 = 2\lceil \frac{n}{2} \rceil \geq n$. Then, $G_B \supseteq K_{1,n}$.

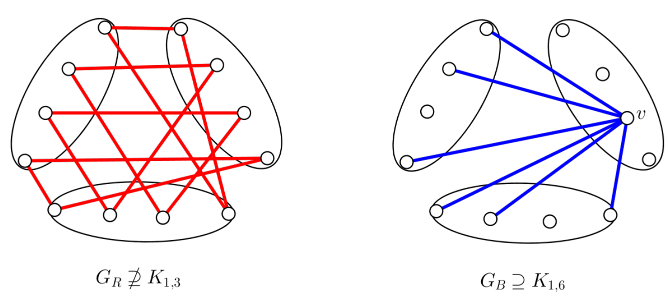

Figure 1. A coloring for $m_3(K_{1,3}, K_{1,6}) = 4$.

For m and n are both even, suppose that $d(v) = m - 1$, for any v in G_R. Then, the sum of the degrees of the vertices of G_R is odd. By *Handshaking* Lemma, it is a contradiction. Then, there is at least one vertex v_1 in G_R such that $d(v_1) = m - 2$. We consider v_1 in G_B for the following two cases.

Case b. For $m \equiv 2 \bmod 4$ and $n \neq 3 \bmod 4$.

$d(v_1) = 2t - m + 2 = 4\lfloor \frac{m+1}{4} \rfloor + 4\lfloor \frac{n}{4} \rfloor + 2 - m + 2 = m - 2 + 4\lfloor \frac{n}{4} \rfloor - m + 4 = 4\lfloor \frac{n}{4} \rfloor + 2 \geq n$.

Case c. For $m \equiv 4 \bmod 4$ and $n \neq 1 \bmod 4$.

$d(v_1) = 2t - m + 2 = 4\lfloor \frac{m-1}{4} \rfloor + 4\lceil \frac{n}{4} \rceil + 2 - m + 2 = m - 4 + 4\lceil \frac{n}{4} \rceil - m + 4 = 4\lceil \frac{n}{4} \rceil \geq n$.

Therefore, there is a star $K_{1,n}$ in G_B, where v_1 as the center. □

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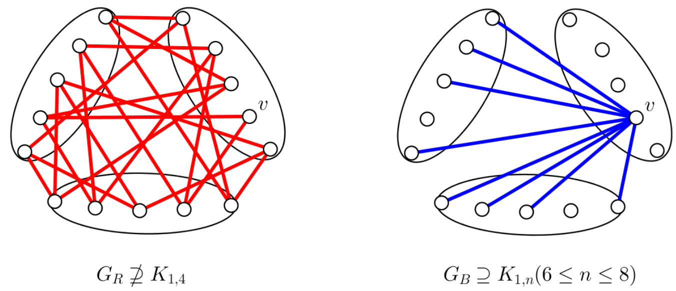

Figure 2. A coloring for $m_3(K_{1,4}, K_{1,n}) = 5, (6 \leq n \leq 8)$.

Theorem 2.4. *For positive integers m and n, we have*

$$m_3(mK_{1,n}, C_3) \geq n\lceil \tfrac{m}{2} \rceil + \lfloor \tfrac{m}{2} \rfloor.$$

Proof. Let $t = n\lceil \frac{m}{2} \rceil + \lfloor \frac{m}{2} \rfloor$. We will show that $m_3(mK_{1,n}, C_3) \geq t$. Let A, B and C be three partite sets in graph $K_{3\times(t-1)}$. We consider a red-blue coloring on the edges of graph $K_{3\times(t-1)} = F_R \oplus F_B$ such that $F_B = K_{t-1,2(t-1)}$, where the first partite set is A and the second partite set is $B \cup C$. This implies that $F_R = K_{2\times(t-1)}$, where the partite sets are B and C. If m is even, then $|V(F_R)| = 2(t-1) = 2(n\lceil \frac{m}{2} \rceil + \lfloor \frac{m}{2} \rfloor - 1) = m(n+1) - 2 < |V(mK_{1,n})|$. Therefore, $F_R \not\supseteq mK_{1,n}$. If $m = 1$, then $F_R = K_{2\times(n-1)}$. It is clear that $F_R \not\supseteq K_{1,n}$. If $m \geq 3$ and m is odd, then $|B| = |C| = \frac{n(m+1)}{2} + \frac{m-3}{2} = \frac{m-1}{2}(n+1) + \frac{n-1}{2}$. Hence, F_R only contains $(m-1)K_{1,n}$. Then, $m_3(mK_{1,n}, C_3) \geq t$. □

Acknowledgement

This research was supported by Research Grant "Penelitian Mandiri" Surat Keputusan No. 438.68/PL1.R7/LT/2019, Politeknik Negeri Bandung, Indonesia.

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