Indonesian Journal of Combinatorics **1** (2) (2017), 47–54

INDONESIAN JOURNAL OF COMBINATORICS

Some results on cordiality labeling of generalized Jahangir graph

R. Hasni[a], S. Matarneh[a], A. Azaizeh[b]

[a]*School of Informatics and Applied Mathematics, University Malaysia Terengganu, 21030 UMT, Kuala Terengganu, Terengganu, Malaysia*
[b]*College of Applied Sciences and Community Service, University of Dammam, KSA*

hroslan@umt.edu.my, s_math_1985_2009@yahoo.com.my

Abstract

In this paper we consider the cordiality of a generalized Jahangir graph $J_{n,m}$. We give sufficient condition for $J_{n,m}$ to admit (or not admit) the prime cordial labeling, product cordial labeling and total product cordial labeling.

Keywords: Generalized Jahangir graph, Prime cordial labeling, Product cordial labeling, Total product cordial labeling
Mathematics Subject Classification: 05C78, 05C38
DOI: 10.19184/ijc.2017.1.2.1

1. Introduction

Let $G = (V, E)$ be the connected, simple and undirected graph with vertex set V and edge set $E(G)$. For standard terminology and notations in Graph Theory, we refer [18]. By a *labeling* we mean any mapping that carries aset of graph elements to a set of numbers (usually positive integers), called *labels*. If the domain of the mapping is the set of vertices or the set of edges, then the labeling is called a *vertex labeling* or (*edge labeling*). If the domain is $V \cup E$ then we call the labeling as *total labeling*. Many labeling schemes have been introduced so far and they are well explored by many researchers. For a dynamic survey on various graph labeling problems, we refer to Gallian [3].

A labeling $f : V(G) \to \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f. If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$

Received: 06 Jan 2017, Revised: 19 May 2017, Accepted: 26 May 2017.

is given by $f^*(e) = |f(u) - f(v)|$. Then $v_f(i)$ is the number of vertices of G having label i under f and $e_f(i)$ is the number of edges of G having label i under f, where $i = 0$ or 1.

Definition 1. A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [2] as a weaker version of graceful labeling and harmonious labeling.

The notion of prime labeling was originated by Entringer and was introduced by Tout et al. [17]. Motivated by the concepts of prime labeling and cordial labeling, a new concept termed as a prime cordial labeling was introduced by Sundaram et al. [8] as follows.

Definition 2. A *prime cordial* labeling of a graph G is a bijection $f : V(G) \to \{1, 2, \ldots, | V(G) |\}$ and the induced edge function $f^* : V(G) \to \{1, 0\}$ is defined by

$$f^*(e = uv) = \begin{cases} 1, & \text{if } gcd(f(u), f(v)) = 1 \\ 0, & \text{if } gcd(f(u), f(v)) > 1 \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. The graph admits a prime cordial labeling is called a *prime cordial graph*.

Many graph families proved to be prime cordial, for example see [8, 12, 13, 14].

In 2004, Sundaram et al. [9] introduced the product cordial labeling of graph.

Definition 3. Let $f : V(G) \to \{0, 1\}$ be a vertex labeling of a graph G that induces an edge labeling function $f^* : E(G) \to \{0, 1\}$ such that $f^*(uv) = f(u)f(v)$ where $uv \in E(G)$. Then f is a *product cordial* labeling if $|v_f(1) - v_f(0)| \leq 1$ and $|e_f(1) - e_f(0)| \leq 1$. A graph G is product cordial if it admits a product cordial labeling.

In 2006, Sundaram et al. [10] introduced the notion of total product cordial labeling of graph.

Definition 4. Let $f : V(G) \to \{0, 1\}$ be a vertex labeling of a graph G that induces an edge labeling function $f^* : E(G) \to \{0, 1\}$ such that $f^*(uv) = f(u)f(v)$ where $uv \in E(G)$. Then f is a *total product cordial* labeling if $| (v_f(1) + e_f(1)) - (v_f(0) + e_f(0))| \leq 1$. A graph G is total product cordial if it admits a total product cordial labeling.

For more results on product cordial and total product cordial, please refer [4, 5, 6, 9, 10, 11].

For $n, m \geq 2$, the generalized Jahangir graph $J_{n,m}$ is a graph on $nm + 1$ vertices, that is, the graph consists of a cycle C_{mn} with one additional vertex which adjacent to a m vertices of C_{mn} at distance n to each other on C_{mn}, see [1, 7]. The following figure shows the graph $J_{n,m}$ for $n = 3$ and $m = 10$.

In this paper, we investigate prime cordial labeling, product cordial labeling and total product cordial labeling of generalized Jahangir graph $J_{n,m}$.

2. Main Results

In this section, we present our main results.

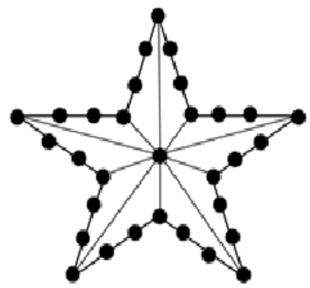

Figure 1. Jahangir graph $J_{3,10}$

Lemma 2.1. *The Jahangir graph $J_{2,m}$, $m \geq 4$ is prime cordial.*

Proof. Let $J_{2,m}, m \geq 4$ be Jahangir graph with the vertex set $V(J_{2,m}) = \{v\} \cup \{v_i : 1 \leq i \leq 2m\}$ and the edge set $E(J_{2,m}) = \{v_iv_{i+1} : 1 \leq i \leq 2m-1\} \cup \{v_{2m}v_1\} \cup \{vv_{2i-1} : 1 \leq i \leq m\}$. Clearly that $|V(J_{2,m})| = 2m + 1$ and $|E(J_{2,m})| = 3m$.

To show that $J_{2,m}$ is a prime cordial, we define a vertex labeling$f : V(J_{2,m}) \to \{1, 2, \cdots, 2m+1\}$ in the following way:

$f(v) = 2$, $f(v_1) = 6$, $f(v_2) = 4$, $f(v_{2m}) = 3$, $f(v_{2m-1}) = 9$, $f(v_{2m-2}) = 5$, $f(v_{2m-3}) = 7$, $f(v_{2m-4}) = 1$

$$f(v_i) = \begin{cases} 2(i+1), & \text{if } 3 \leq i \leq \lfloor \frac{2m-1}{2} \rfloor \\ 4m - 2i + 1, & \text{if } \lfloor \frac{2m-1}{2} \rfloor + 1 \leq i \leq 2m - 5 \end{cases}$$

We have $|e_f(0)| = \lfloor \frac{3m}{2} \rfloor$ and $|e_f(1)| = \lceil \frac{3m}{2} \rceil$. Then $|e_f(0) - e_f(1)| \leq 1$. Hence, the Jahangir graph $J_{2,m}$ is prime cordial.

This completes the proof. □

Theorem 2.1. *The Jahangir graph $J_{n,m}$, $n > 2$, $m > 3$ is prime cordial.*

Proof. Let $J_{n,m}, n > 2, m > 3$ be Jahangir graph with the vertex set $V(J_{n,m}) = \{v\} \cup \{v_i : 1 \leq i \leq mn\}$ and the edge set $E(J_{n,m}) = \{v_iv_{i+1} : 1 \leq i \leq mn-1\} \cup \{v_{mn}v_1\} \cup \{vv_{n(i-1)+1} : 1 \leq i \leq m\}$. Clearly that $|V(J_{n,m})| = mn + 1$ and $|E(J_{n,m})| = n(m + 1)$.

To show that $J_{n,m}$ is prime cordial, we define vertex labeling $f : V(J_{n,m}) \to \{1, 2, \ldots, mn+1\}$ as follows:

$$f(v) = 2, f(v_1) = 6, f(v_2) = 4, f(v_{mn}) = 3$$

$$f(v_{mn-1}) = \begin{cases} 1, & \text{for } m, n \text{ are odd} \\ 9, & \text{for otherwise} \end{cases}$$

$$f(v_{mn-2}) = \begin{cases} 5, & \text{for } m,n \text{ are even} \\ & \text{or } m,n \text{ are odd} \\ 1, & \text{for otherwise} \end{cases}$$

$$f(v_{mn-3}) = \begin{cases} 7, & \text{if } m,n \text{ are even} \\ & \text{or } m,n \text{ are odd} \\ 5, & \text{for otherwise} \end{cases}$$

$$f(v_{mn-4}) = \begin{cases} 1, & \text{if } m,n \text{ are even} \\ 9 & \text{if } m,n \text{ are odd} \\ 7, & \text{for otherwise} \end{cases}$$

$$f(v_i) = \begin{cases} 2(i+1), & \text{if } 3 \leq i \leq \lfloor \frac{nm-1}{2} \rfloor \\ 2mn - 2i + 1, & \text{if } \lfloor \frac{nm-1}{2} \rfloor + 1 \leq i \leq mn - 5. \end{cases}$$

We have

$$|e_f(0)| = \begin{cases} \frac{(m(n+1)-1)}{2}, & \text{if } m \text{ are even} \\ & \text{and } n \text{ are odd} \\ \frac{(m(n+1))}{2}, & \text{for otherwise} \end{cases}$$

$$|e_f(1)| = \begin{cases} \frac{(m(n+1)+1)}{2}, & \text{if } m \text{ are even} \\ & \text{and } n \text{ are odd} \\ \frac{(m(n+1))}{2}, & \text{for otherwise} \end{cases}$$

It is easy to show that $|e_f(0) - e_f(1)| \leq 1$. Hence, the Jahangir graph $J_{n,m}$ is prime cordial. This completes the proof. □

The following figure illustrates the prime cordial labeling of graph $J_{3,5}$.

Theorem 2.2. *The Jahangir graph $J_{n,m}$ is product cordial with $n \geq 2$, $m \geq 3$, m is odd and n is even.*

Proof. Let $J_{n,m}$ with n is even and $n \geq 2$, m is odd and $m \geq 3$, be Jahangir graph with the vertex set $V(J_{n,m}) = \{v\} \cup \{v_i : 1 \leq i \leq mn\}$ and the edge set $E(J_{n,m}) = \{v_iv_{i+1} : 1 \leq i \leq mn-1\} \cup \{v_{mn}v_1\} \cup \{vv_{n(i-1)+1} : 1 \leq i \leq m\}$. Clearly that $|V(J_{n,m})| = mn + 1$ and $|E(J_{n,m})| = m(n + 1)$.

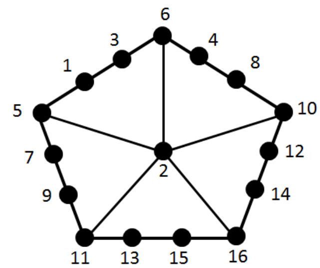

Figure 2. Prime cordial labeling of $J_{3,5}$

To show that $J_{n,m}$ is product cordial, define a vertex labeling $f : V(J_{n,m}) \to \{0,1\}$ in the following way:

$$f(v) = 1$$

$$f(v_i) = \begin{cases} 1, & \text{if } 1 \leq i \leq \frac{nm}{2} \\ 0, & \text{if } \frac{nm}{2} + 1 \leq i \leq mn \end{cases}$$

From the above labeling, we can see that $v_f(1) = \frac{mn+2}{2}$, $v_f(0) = \frac{mn}{2}$, $e_f(1) = \frac{m(n+1)-1}{2}$, $e_f(0) = \frac{m(n+1)+1}{2}$. Hence $|v_f(1) - v_f(0)| = 1$ and $|e_f(1) - e_f(0)| = 1$. Therefore the graph $J_{n,m}$ is product cordial.

This completes the proof. □

Figure 3 below illustrates the product cordial labeling of graph $J_{2,5}$.

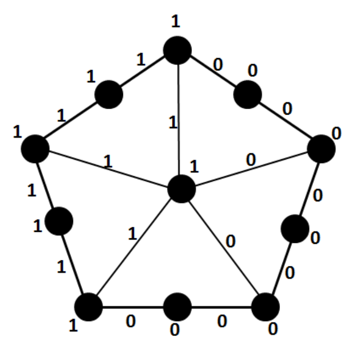

Figure 3. Product cordial labeling of $J_{2,5}$

In Theorem 2.2, the graph $J_{n,m}$ is product cordial labeling for $n \geq 2$, $m \geq 3$, m is odd and n is even. We have tried to find the product cordial labeling of $J_{n,m}$ for all values of m and n but so far without success. So we pose the following open problem.

Problem 1. *Determine product cordial labeling of the Jahangir graph $J_{n,m}$ for all m and n.*

Theorem 2.3. *The Jahangir graph $J_{n,m}$, $n \geq 2$, $m \geq 3$ is total product cordial.*

Proof. Let $J_{n,m}$, $n \geq 2$, $m \geq 3$ be a Jahangir graph with the vertex set $V(J_{n,m}) = \{v\} \cup \{v_i : 1 \leq i \leq mn\}$ and the edge set $E(J_{n,m}) = \{v_i v_{i+1} : 1 \leq i \leq mn-1\} \cup \{v_{mn} v_1\} \cup \{v v_{n(i-1)+1} : 1 \leq i \leq m\}$. Clearly that $|V(J_{n,m})| = mn + 1$ and $|E(J_{n,m})| = m(n+1)$.

To show that $J_{n,m}$ is total product cordial, define a vertex labeling $f : V(J_{n,m}) \to \{0,1\}$ in the following way:

Case 1: m and n are odd.

$$f(v) = 1, f(v_{mn-1}) = 1$$

$$f(v_i) = \begin{cases} 1, & \text{if } 1 \leq i \leq \frac{nm-1}{2} \\ 0, & \text{if } \frac{nm+1}{2} \leq i \leq mn, i \neq mn-1 \end{cases}$$

We have $|v_f(1)| = \lceil \frac{mn+2}{2} \rceil$, $|v_f(0)| = \lfloor \frac{mn}{2} \rfloor$, $|e_f(1)| = \frac{m(n+1)-2}{2}$, $|e_f(0)| = \frac{m(n+1)+2}{2}$. It is easy to see that $|(v_f(1) + e_f(1)) - (v_f(0) + e_f(0))| \leq 1$. Hence the graph $J_{n,m}$ is total product cordial.

Case 2: m and n are not odd.

$$f(v) = 1$$

$$f(v_i) = \begin{cases} 1, & \text{if } 1 \leq i \leq \frac{nm}{2} \\ 0, & \text{if } \frac{nm+2}{2} \leq i \leq mn \end{cases}$$

We have $|v_f(1)| = \frac{mn+2}{2}$, $|v_f(0)| = \frac{mn}{2}$, $|e_f(1)| = \lfloor \frac{m(n+1)-1}{2} \rfloor$, $|e_f(0)| = \lceil \frac{m(n+1)+1}{2} \rceil$. It is easy to see that $|(v_f(1) + e_f(1)) - (v_f(0) + e_f(0))| \leq 1$. Hence the graph $J_{n,m}$ is total product cordial.

This completes the proof. □

Figure 4 shows the total product cordial labeling of graph $J_{4,5}$.

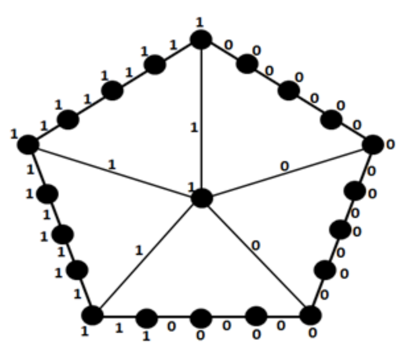

Figure 4. Total product cordial labeling of graph $J_{4,5}$

In [15, 16], Vaidya and Barasara introduced an edge product cordial labeling and a total edge product cordial labeling of graph G. Thus, we propose the following problem.

Problem 2. *Determine edge product cordial labeling and total edge product cordial labeling of the Jahangir graph* $J_{n,m}$ *for* $n, m \geq 2$.

Acknowledgement The authors would like to thank the referee for his/her valuable comments which improved the paper.

[1] K. Ali, E.T. Baskoro and I. Tomescu, On the Ramsey number of paths and Jahangir graph $J_{3,m}$, The 3rd International Conference on 21st Century Mathematics, GC University Lahore, Pakistan, 2007.

[2] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23(1987) 201–207.

[3] J.A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics 5, vol. 18, p. DS6, 2011.

[4] Z.B. Gao, G.Y. Sun, Y.N. Zhang, Y. Meng and G.C. Lau, Product cordial and total product cordial labelings of P_{n+1}^m, Journal of Discrete Mathematis, Volume 2015, Article ID 512696, 6 pages.

[5] H. Kwong, S.M. Lee and H.K. Ng, On product cordial index sets and friendly index sets of 2-regular graphs and generalized wheels, Acta Math. Sinica, English Series, 28(4)(2012), 661–674.

[6] H. Kwong, S.M. Lee and H.K. Ng, On product cordial index sets of cylinders, Congressus Numerantium, 206(2010), 139–150.

[7] D. A. Mojdeh and A. N. Ghameshlou domination in Jahangir graph $J_{2,m}$ Int. J. Contemp. Math. Sciences, 2(24)(2007), 1193–1199.

[8] M. Sundaram, R. Ponraj and S. Somasundram, Prime cordial labeling of graphs, Journal of Indian Academy of Mathematics, 27(2)(2005), 373-390.

[9] M. Sundaram, R. Ponraj and S. Somasundram, Product cordial labeling of graphs, Bulletin of Pure and Applied Science (Mathematics and Statistics), 23(2004), 155–163.

[10] M. Sundaram, R. Ponraj and S. Somasundram, Total product cordial labeling of graphs, Bulletin of Pure and Applied Science (Mathematics and Statistics), 25(1)(2006), 199–203.

[11] M. Sundaram, R. Ponraj and S. Somasundram, Some results on total product cordial labeling of graphs, Indian Academy of Mathematics, 28(2)(2006), 309–320.

[12] S.K. Vaidya and P.L. Vihol, Prime cordial labeling for some cycle related graphs, International Journal of Open Problems in Computer Science and Mathematics, 3(5)(2010), 223–232.

[13] S.K. Vaidya and P.L. Vihol, Prime cordial labeling for some graphs, Modern Applied Sciences, 4(8)(2010), 119–126.

[14] S.K. Vaidya and N.H. Shah, Some new results on prime cordial labeling, ISRN Combinatorics, Volume 2014, Article ID 607018, 9 pages.

[15] S.K. Vaidya and C.M. Barasara, Edge product cordial labeling of graphs, Journal of Mathematical and Computational Science, 2(5)(2012), 1436–1450.

[16] S.K. Vaidya and C.M. Barasara, Total edge product cordial labeling of graphs, Malaya Journal of Matematik, 3(1)(2013), 55–63.

[17] A. Tout, A.N. Dabboucy and K. Howalla, Prime labeling of graphs, National Academy Science Letter, 11(1982), 365–368.

[18] D.B. West, Introduction to Graph Theory, 2nd Edition, Prentice Hall, USA, 2001.