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Exploring the power of graph theory in Hadron theory: From bound states to quark systems

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Abstract

The article discusses how graph theory has been utilized in hadron theory for high energy interactions in recent years. The paper emphasizes the significance of a visual perspective throughout the discussion and explores how graph theory can aid in creating various systems such as bound state systems. Additionally, the paper delves into how graph theory has been used to develop few-body quark systems and how it can connect with adjacency and incidence matrices in the graph theory by providing applications of how these fundamental principles have been applied to topics ranging from the hadronic bound states in the quantum models.

Keywords: Graph theory, Quantum theory
Mathematics Subject Classification : 05C90

1. Introduction

Particle physics, specifically astrophysics, aims to comprehend the laws of nature through the analysis of basic particles in controlled or natural environments. The standard model clarifies the behavior of elementary particles like quarks and leptons, along with the strong, weak, and electromagnetic forces. Researchers carry out experiments to measure these particles and utilize statistical approaches to assess various theories. Unfortunately, experimental data only provides restricted knowledge about the physical process, particularly in terms of time and space [34].

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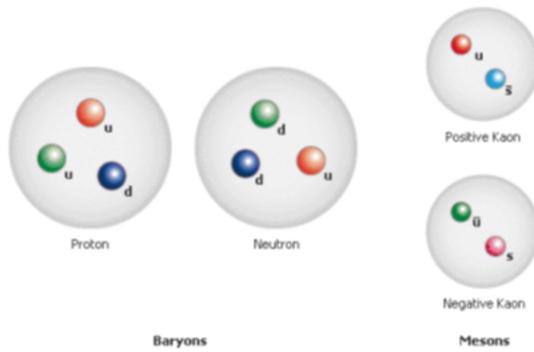

Figure 1. The baryons and mesons are represented as quark states [27]

In the early 1960s, accelerators were able to achieve higher energies which led to the discovery of over a hundred new particles, revealing that matter is made up of small particles called quarks. Quarks have six different types: up (u), bottom (s), charm (c), strange (s), top (t), and bottom (b), and their antimatter counterparts are known as antiquarks, represented by a line over the letter symbol. When quarks and antiquarks combine, they create mesons, while quarks combining with quarks make bigger particles called baryons as shown in Fig. 1. Protons and neutrons in an atom's nucleus are examples of baryons. Mesons contain positive and negative kaons as shown in Fig. 2. One successful theory that explains the strong interaction between quarks, mediated by gluons, is quantum chromodynamics (QCD) theory [27]. QCD is distinguished by three main features - color confinement, asymptotic freedom, and chiral symmetry breaking, and various approaches have been developed to study it. Perturbative QCD, lattice QCD, effective theories, 1/n expansion, and QCD sum rules are among the techniques used to understand QCD.

Lattice QCD is defined on a discrete Euclidean space-time grid. Lattice QCD maintains the core characteristics of QCD as no additional parameters or field variables are introduced in this discretization. Lattice QCD may be used for two items. Initially, the discrete space-time lattice functions as a regularization scheme without perturbation. There are no infinities for finite values of the lattice spacing a, which offers an ultraviolet cutoff. Renormalized physical quantities also have a finite well-behaved limit, denoted by $a \to 0$. Hence, in theory, using lattice regularization, one might perform all the common perturbative calculations; nevertheless, these calculations are far more difficult and offer no benefit over those performed using a continuum scheme. The most important use of converting QCD to a space-time lattice is the ability to simulate LQCD on a computer using techniques similar to those used for statistical mechanics systems [17].

In the field of hadron physics, chiral symmetry breaking is a significant occurrence that plays a crucial role in understanding the characteristics of hadrons. To efficiently model low-energy features from QCD, which is the fundamental theory of strong interactions, the creation of effective models has been driven by the challenges associated with this process due to QCD's simplicity and efficacy in representing hadrons at low energies [36]. The linear sigma model has been utilized to model strong nuclear interactions [16], which was originally proposed as a theory for pion-nucleon

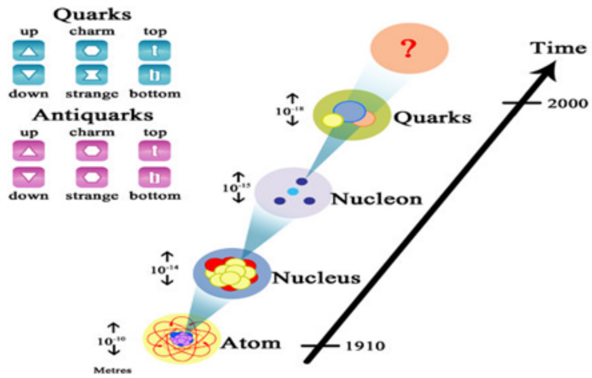

Figure 2. From Atom to the quark states1

interactions during the 1960s. At present, the model is considered an effective representation of the low-energy phase of quantum chromodynamics at zero-temperature [[1], [10], [32], [33]]. Refs. [[2] - [7]] suggest modifying the model to precisely depict the characteristics of baryon.

2. The Graph theory in the particle physics

Leonhard Euler first began studying group theory in the 18th century and that it was further developed by Gauss and J. J Sylvester in the 19th century. Group theory (GT) has since become an important area of study in mathematics, with applications in various fields such as physics, chemistry, and cryptography. The development of interdisciplinary activity has helped to expand the applications of group theory and make it relevant in many areas of research. Particularly, relevant in QFT, Feynman diagram techniques and determination of the eightfold way.

With the investigation of GT, researchers pursue the understanding of basic particle physics interactions and connections in exotic states. The usage of GT to represent Feynman diagrams or predict novel and exotic bound states is then taken into consideration. It is crucial and helpful to apply GT for the investigation of the hadronic universe. The application of mathematical ideas to the resolution of physical issues is what is known as theoretical particle physics. This encompasses almost every branch of physics, most notably theoretical particle physics. Solutions to bound state difficulties and particle interaction in high energy physics are dealt with by fields such GT. As a result, numerous divisions work along with the graph theory and its applications in fields like hadronic physics. Bounding states, the quark cluster model, and interaction diagrams are some of the tools the group and graph theories use to keep particle physics, in the broadest terms, at the junction of two major fields. Thus, GT has a crucial role to play in particle physics. The graph of a constitutional hadronic system has nodes (vertices) that represent quarks and gluons and lines (edges) that indicate bound states or virtual interactions based on QFT in either hybrid space-time or conventional space-time. Hence, new significant GT applications in theoretical particle physics are identified. A combination of nodes-line-path-loop in graph model may be used to explore the whole set of all possible hadronic cluster or few body gluon bound states, exotic baryons, or exotic meson, see [22]. In simple terms, the jets produced by pure hadronic interaction (QCD jets) are difficult to distinguish from other jets due to their wide range and act as a constant background. Particle flow reconstruction is used to analyze these events by identifying particle candidates [35]. The particle candidates are then shown on a graph and different techniques are employed for further analysis as described in Refs. [[9], [11], [21], [23], [25], [26], [30], [31]].

The study discussed in Ref. [11] utilizes a message passing architecture and fully connected graph to train the adjacency matrix. The proposed approach was tested by comparing different constructs of directed and undirected graphs, and the findings indicated that it led to enhanced classification of jets that come from the decay of a W boson as well as QCD jets. The authors suggest further research on physics-based inductive biases to enhance the learning of the adjacency matrix, which has been connected to learning attention as mentioned in Ref. [26].

The authors in Ref. [21] used the edgeconv method introduced in Ref. [38] to create a point cloud architecture for jet tagging. The graph connectivity of the architecture is determined dynamically based on the node neighborhoods' distance in either the input space or an intermediate latent space when graph layers are stacked. To account for particle permutation invariance, the architecture averages contributions from connected neighbors. The model's performance for tasks such as quark/gluon discrimination and top tagging is reported to be superior to other architectures previously studied. The model's learned edge function is limited to using the node feature and the feature difference between the node and its connected node as its input. In Ref. [9], the authors utilized the same model design to analyze semi-visible jets produced by the decay of unknown dark hadrons. Their approach surpasses image-based neural networks and models that rely on physical assumptions [12]. Through their findings, they highlight that implementing this method improves the accuracy of detecting dark matter by a significant degree.

The interaction network architecture was adapted by the authors of [[30], [31]] for graph categorization. They utilized a fully connected graph over the particles of a jet and primary vertices of an event to extract a graph category through message passing. The model outperforms other non-graph-based architectures in multiclass categorization and in tagging jets that refers to the occurrence of events resulting from the decay of Higgs bosons into two b quarks, despite efforts to remove the effects of mass correlation using appropriate methods [13]. However, there may be some computation performance issues with running the model for predictions. The authors suggest that the performance could be improved with a native implementation instead of the model obtained from a format conversion between major frameworks.

Two complementary field-programmable gate array (FPGA) implementations of a graph neural network (GNN) for charged particle tracking at the LHC. The conversion of the trained model, specified using PyTorch Geometric, into high-level synthesis (HLS) code is achieved automatically using a custom converter integrated into hls4ml, a source-to-source compiler. Namely, the GNN classifies track segments as true or false based on a graph constructed from the positions of hits in the tracking detector [14]. To reconstruct the higgs jet in collider events. The authors in Ref. [15] use a graphic convolutional network (GCN) with focal loss by representing each event as a point cloud. This approach offers superior performance in higgs tagging efficiency and reconstruction accuracy compared to traditional methods relying on jet substructure information, as demonstrated by prior works. In Ref. [37], the authors have developed a model that can predict the mass of a halo based on information about the positions, velocities, stellar masses, and radii of the galaxies within it. To do this, they have used a type of machine learning algorithm called graph neural networks (GNNS), which are well-suited to handling sparse and irregular data (like the distribution of galaxies within a halo). Also, GNNS plays a basic role molecular and property prediction tasks and fixed spatial sizes when using deep neural networks to model electromagnetic fields. [39], [24]. The work [19] explains the importance of accurate and fast simulation of particle physics processes, particularly in detecting rare phenomena that can expand our knowledge of new interactions. It also highlights the challenges faced by the high-energy physics community in simulating particle interactions with detectors due to their time-consuming and computationally expensive nature. The work then proposes the use of machine learning approaches to provide faster solutions while maintaining a high level of accuracy in simulating high-energy physics collisions. Specifically, it discusses a graph generative model that can effectively reconstruct LHC events, which could lead to full detector level fast simulation for HL-LHC. In Ref. [23], a new neural approach is presented for reconstructing hierarchical interactions in rooted tree graphs. This approach utilizes a novel representation called the lowest common ancestor generations (lcag) matrix, which allows for the learning of a tree's structure from its leaves without any prior assumptions. The lcag matrix is the first end-to-end trainable solution that can learn the hierarchical structure of trees of varying sizes directly, without relying on the adjacency matrix. The use of lcag is demonstrated in predicting simulated particle physics decay structures using both transformer and neural relational inference encoders, for trees up to 6 leaves and up to 10 in the simulated dataset, with a maximum tree-depth of 8. This work is particularly relevant to high-energy particle physics, as it deals with the hierarchical tree structures formed by the final products of particle decay, which have a large combinatorial space of possible trees and are therefore intractable to analyze.

In Ref. [20] describes a newly developed algorithm that is capable of distinguishing jets that come from tau lepton decays from those that come from quarks or gluons. The algorithm doesn't rely on a tau lepton reconstruction process. Rather, it treats the jets as diverse graphs, with nodes represented by tracks and energy clusters. It then trains a graph neural network to identify tau jets from other jets. The algorithm explores different attributed graph representations and different GNN architectures. The proposed method uses differential track and energy cluster information as node features and a heterogeneous sequentially-biased encoding for the inputs to final graph-level classification. In Ref. [29] describes the use of machine learning methods, specifically graph neural networks, to perform jet tagging at the CERN LHC. Jet tagging involves inferring the origin of a jet based on its final-state particles. To accomplish this task, the particle net model is employed, which treats the jets as collections of points and includes connections between the particles that can be learned. The layer wise-relevance propagation technique is used to identify relevant edge connections in the decision-making process of the model. The model is seen to alter the pattern of important connections between individual clusters of particles, which are referred to as subjets. Signal jets originating from top quarks have a different subjet connection distribution compared to background jets originating from lighter quarks and gluons. This implies that the model is determining the origin of jets using conventional jet substructure observables, including the number of prongs.

In Ref. [28] explores the use of geometric deep learning on Feynman diagrams to accurately and efficiently predict matrix elements. The research employs the graph attention layer and demonstrates high accuracy in making predictions up to 3 significant figures with less than 200 epochs

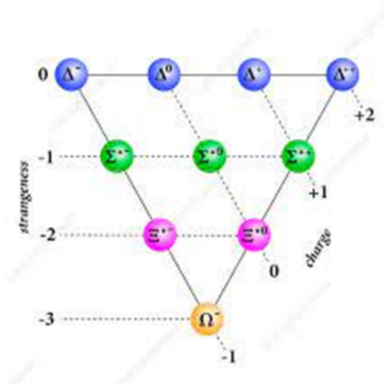

Figure 3. Illustrates the grouping of three u, d or s-quarks that have a combined spin of 3/2, which is referred to as the baryon decuplet [http://en.wikipedia.org].

of training. By enabling the creation of Feynman diagrams with effective particles to describe non-perturbative computations, this method may have major implications for quantum field theory. Overall, this work highlights the potential of machine learning to advance our understanding of particle physics. In Ref. [18] discusses the problem of learning the pairwise interaction law in interacting particle systems, which is essential for fully comprehending such systems but remains an open challenge due to the complexity of the underlying physical laws, so the proposed technique is to modify the graph network framework, which has a node component to describe particle-level dynamics and an edge part to learn pairwise interaction. By only being trained to anticipate particle acceleration, the algorithm's deterministic operator in the node part enables exact inference of the pairwise interactions that are compatible with underlying physical laws. The developed methodology for inferring particle interactions has several advantages over existing approaches. It performs better in terms of satisfying Newton's laws, generalizing to larger systems, and handling noise. This methodology can improve understanding and discovery of particle interaction laws, leading to better design of materials with specific properties. In Ref. [8] explains how partition pooling can be used in convolutional graph networks for effective event reconstruction and classification in particle physics. It also highlights the benefits of this pooling scheme, such as reduced computational resources, allowing for deeper networks and more extensive hyperparameter optimizations. To show its applicability, a convolutional graph network with partition pooling was constructed and was found to yield improved performance and be less susceptible to overfitting. In Fig. 3, we presented baryon decuplet by undirected adjancy matrix in Table 1. Also, the octet of light spin 1/2 baryons in Table 2. In Table 3, we displayed Feynman diagram for general features of the scattering process by directed incidence matrix.

3. Summary and Conclusion

This work presents theoretical information on the structure and interactions of particles using GT. The article defines interactions and determines their Hamiltonian based on examining node-line behavior in QFT graphs. The results demonstrate that using GT to analyze particle inter-

Table 1. Undirected adjacency matric in graph theory for Fig. 3

V	$\wedge^-$	$\wedge^0$	$\wedge^+$	$\wedge^{++}$	$\sum^-$	$\sum^0$	$\sum^+$	Ξ^-	Ξ^0	Ω^-
$\wedge^-$	0	1	0	0	1	0	0	0	0	0
$\wedge^0$	1	0	1	0	0	1	0	0	0	0
$\wedge^+$	0	1	0	0	0	0	1	0	0	0
$\wedge^{++}$	0	0	1	0	0	0	1	0	0	0
$\sum^-$	1	0	0	0	0	1	0	1	0	0
$\sum^0$	0	1	0	0	1	0	1	0	1	0
$\sum^+$	0	0	1	1	0	1	0	0	1	0
Ξ^-	0	0	0	0	1	0	0	1	1	1
Ξ^0	0	0	0	0	0	1	1	0	0	1
Ω^-	0	0	0	0	0	0	0	1	1	0

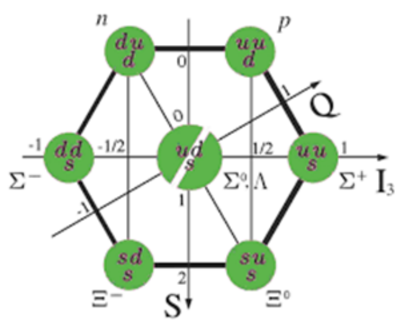

Figure 4. The octet of light spin-1/2 baryons [http://en.wikipedia.org]

Table 2. Undirected adjacency matric in graph theory for Fig. 4

V	n	p	$\sum^-$	$\sum^0$	$\sum^+$	Ξ^-	Ξ^0	
n	0	1	1	1	0	0	0	0
p	1	0	0	0	1	1	0	0
$\sum^-$	1	0	0	1	0	1	1	0
$\sum^0$	1	0	1	0	1	0	0	1
$\sum^+$	0	1	0	1	0	1	0	1
Ξ^-	0	0	1	0	0	0	0	1
Ξ^0	0	0	0	1	1	1	1	0

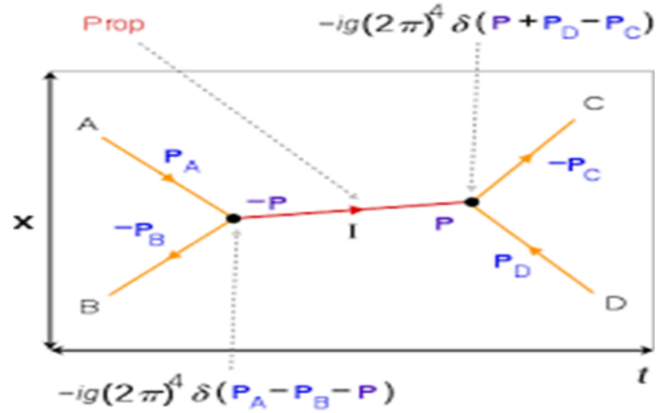

Figure 5. Feynman Diagram for general features of the scattering process $A + B \to C + D$

Table 3. Directed incidence matrix for Feymanm digram of Fig. 5

	P_A	$-P_B$	I	$-P_C$	P_D
$P-$	1	-1	-1	0	0
P	0	0	1	-1	1

actions can help identify bound states, hadronic systems, and quark family lines. Unlike Feynman diagrams that only describe events in space-time, GT can describe specific objects through nodes, path lines, or path loops, and can be represented by adjacency and incidence metrics. However, Feynman diagrams are unable to account for currently existing particles.

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