Indonesian Journal of Combinatorics **9** (1) (2025), 13–20

INDONESIAN JOURNAL OF COMBINATORICS

Local edge antimagic coloring for chain of path and cycle

Yosua Walfried[a], Ivana Joice Chandra[a], Denny Riama Silaban[*a,b]

[a]*Departments of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok - Indonesia*
[b]*Center for Research Collaboration on Graph Theory and Combinatorics, Indonesia*

yosua.walfried@ui.ac.id, ivana.joice@ui.ac.id, denny@sci.ui.ac.id

[*]*Corresponding author*

Abstract

Let $G = (V, E)$ be a simple connected graph with vertex set V and edge set E. A local edge antimagic labeling of G is a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ where the weights of any two adjacent edges of G are distinct. The weight of an edge uv is defined as $w(uv) = f(u) + f(v)$. By assigning the color $w(uv)$ to each edge $uv \in E(G)$, we obtained a proper local edge antimagic coloring of G. The minimum number of colors required for edge coloring induced by the local edge antimagic labeling is called the local antimagic chromatic index of G. In this article, we give the exact value of the local antimagic chromatic index for chains of path and cycle graphs.

Keywords: Cycle, local antimagic chromatic index, local edge antimagic labeling, path.
Mathematics Subject Classification: 05C78; 05C69

1. Introduction

An antimagic labeling was introduced by Hartsfield and Ringel in 1990 [6] and in 2017, Arumugam et al. [2] proposed the local version of antimagic labeling that induced a proper vertex coloring of a graph. In 2017, Agustin et al. [1] gave the variation of local antimagic labeling called the local edge antimagic labeling that induced an appropriate edge coloring of a graph.

Let $G = (V, E)$ be a simple connected graph with a vertex set V and an edge set E. A local edge antimagic labeling of G is a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ if the weight of any adjacent edges is different. The weight of the edge uv is $w(uv) = f(u) + f(v)$. By assigning edge weight as the edge color, the minimum number of colors needed for the edge coloring of

Received: 12 February 2025, Accepted: 04 June 2025.

G is called the local antimagic chromatic index of G, $\chi'_{lea}(G)$. It is clear that $\chi'_{lea}(G) \geq \chi'(G)$ where $\chi'(G)$ is the chromatic index of G. By Vizing's Theorem (Theorem 1.1) we have that the maximum degree of G is the general lower bound for the local antimagic chromatic index of G, that is $\chi'_{lea}(G) \geq \Delta(G)$.

Theorem 1.1 ([5]). *For any finite, simple graph $G, \Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$.*

In 2017, Arumugam et al. [2] not only give some properties of the local antimagic chromatic number of a graph, but also investigate the number for some classes of graphs, including path, cycle, and complete graph. Their work provided initial results on the chromatic number required for local antimagic edge coloring and sparked interest in exploring other graph structures. Furthermore, in 2019, Bensmail et al. [3] examined trees and established lower bounds on the number of colors necessary for such graphs, contributing to the understanding of how local antimagic labeling operates on non-cyclic graphs. In addition, a study by Choi et al. [4] explored local antimagic edge colorings in generalized graph families, including connected and disconnected graphs.

In subsequent studies, researchers explored other classes of graphs, such as bipartite graphs, cubic graphs, and planar graphs. For example, Lee et al. [7] studied the behavior of local antimagic edge colorings in planar graphs and found that certain planar graphs have significantly different chromatic properties compared to non-planar graphs.

In this paper, we give the local antimagic chromatic index of a chain of graphs involving paths and cycles. Let $C_a = y_1y_2...y_ay_1$ for $a \geq 3$ and $P_b = x_1x_2...x_b$ for $b \geq 2$. For any positive integer s, the graph sC_aP_b is constructed by taking s copies of C_a and $s-1$ copies of P_b and identifying x_1 from the i-th copy of P_b with y_1 from the i-th copy of C_a and x_b from the i-th copy of P_b with $y_{\frac{a}{2}+1}$ from the $(k+1)$-th copy of C_a, where $1 \leq i \leq s-1$. Meanwhile, the graph sP_bC_a is constructed by taking s copies of P_b and $s-1$ copies of C_a and identifying $y_{\frac{a}{2}+1}$ from the i-th copy of C_a with x_b from the i-th copy of P_b and y_1 from the i-th copy of C_a with x_1 from the $(k+1)$-th copy of P_b, where $1 \leq i \leq s-1$. Here, we show that for $a \geq 4$ and $b \geq 2$ be even integers, $\chi'_{lea}(sC_aP_b) = \chi'_{lea}(sP_bC_a) = 3$.

2. Main Results

In this section, we determine that the local antimagic chromatic index of the chain of path and cycle graph sC_aP_b and sP_bC_a for both b and a is even.

Theorem 2.1. *Let $a \geq 4$ and $b \geq 2$ be even integers. The local antimagic chromatic index of sC_aP_b is $\chi'_{lea}(sC_aP_b) = 3$.*

Proof. The graph sC_aP_b is a connected graph with vertex set $V(sC_aP_b) = \{x_{k,l,1} : 1 \leq k \leq a, 1 \leq l \leq s\} \cup \{x_{k,l,2} : 1 \leq k \leq b-2, 1 \leq l \leq s-1\}$ and edge set $E(sC_aP_b) = \{x_{k,l,1}x_{k+1,l,1} : 1 \leq k \leq a-1, 1 \leq l \leq s\} \cup \{x_{a,l,1}x_{1,l,1} : 1 \leq l \leq s\} \cup \{x_{k,l,2}x_{k+1,l,2} : 1 \leq k \leq b-3, 1 \leq l \leq s-1\} \cup \{x_{1,l,1}x_{1,l,2} : 1 \leq l \leq s-1\} \cup \{x_{b-2,j,2}x_{\frac{a}{2}+1,j,1} : 1 \leq l \leq s-1\}$. Thus, the number of vertices is $|V(sC_aP_b)| = sa + (s-1)(b-2)$ and the number of edges is $|E(sC_aP_b)| = sa + (s-1)(b-1)$.

In Figure 1 we have the graph sC_aP_b and the vertex.

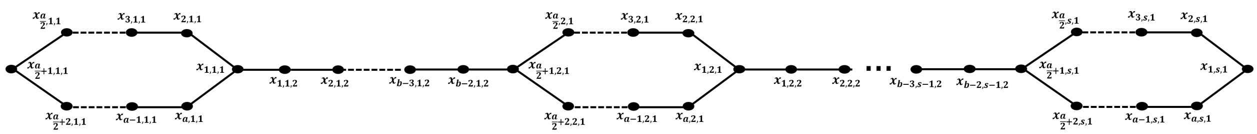

Figure 1: graph sC_aP_b.

Case 1. $a \equiv 0 \pmod 4$.

Define a bijection $f: V(sC_aP_b) \to \{1, 2, 3, \ldots, |V(sC_aP_b)|\}$ to be a local edge antimagic labeling for sC_aP_b as follows.

$$f(x_{k,l,1}) = \begin{cases} \frac{a}{2} - (k-1) + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, \ k \le \frac{a}{2}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) - \left(\frac{a}{2} - k\right) - \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is even}, \ k \le \frac{a}{2}, \ l \text{ is odd}, \\ k - \frac{a}{2} + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, \ k > \frac{a}{2}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) + \left(\frac{a}{2} + 1 - k\right) - \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is even}, \ k > \frac{a}{2}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) - \left(\frac{2a+b-2}{2} - k\right) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ k \le \frac{a}{2}, \ l \text{ is even}, \\ \left(\frac{2a+b-2}{2}\right) - (k-1) + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ k \le \frac{a}{2}, \ l \text{ is even}, \\ sa + (s-1)(b-2) - \left(\frac{b-2}{2}\right) - (k-1) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ k > \frac{a}{2}, \ l \text{ is even}, \\ \left(\frac{b-2}{2}\right) + k + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ k > \frac{a}{2}, \ l \text{ is even}, \end{cases}$$

$$f(x_{k,l,2}) = \begin{cases} \frac{a}{2} + \frac{k}{2} + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is even}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) - \left(\frac{a}{2} + \frac{k-1}{2}\right) - \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) - \left(a + \left(\frac{b-2}{2}\right)\right) - \left(\frac{k-2}{2}\right) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ l \text{ is even}, \\ \left(a + \left(\frac{b-2}{2}\right)\right) + \left(\frac{k+1}{2}\right) + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ l \text{ is even}. \end{cases}$$

The edge weights are as follows.

$$w(x_{k,l,1}x_{k+1,l,1}) = \begin{cases} sa + (s-1)(b-2) + 2, & k < \frac{a}{2}, k \text{ is odd}, \ l \text{ is odd}, \\ sa + (s-1)(b-2), & k < \frac{a}{2}, k \text{ is even}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) + 2, & k < \frac{a}{2}, k \text{ is even}, \ l \text{ is even}, \\ sa + (s-1)(b-2), & k < \frac{a}{2}, k \text{ is odd}, \ l \text{ is even}, \\ sa + (s-1)(b-2) + 1, & k = \frac{a}{2} \\ sa + (s-1)(b-2) + 2, & k > \frac{a}{2}, k \text{ is even}, \ l \text{ is odd}, \\ sa + (s-1)(b-2), & k > \frac{a}{2}, k \text{ is odd}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) + 2, & k > \frac{a}{2}, k \text{ is odd}, \ l \text{ is even}, \\ sa + (s-1)(b-2), & k > \frac{a}{2}, k \text{ is even}, \ l \text{ is even}, \end{cases}$$

$$w(x_{a,l,1}x_{1,l,1}) = sa + (s-1)(b-2) + 1,$$

$$w(x_{k,l,2}x_{k+1,l,2}) = \begin{cases} sa + (s-1)(b-2) + 1, & k \text{ is odd}, \ l \text{ is odd}, \\ sa + (s-1)(b-2), & k \text{ is even}, \ l \text{ is odd}, \\ sa + (s-1)(b-2) + 1, & k \text{ is odd}, \ l \text{ is even}, \\ sa + (s-1)(b-2) + 2, & k \text{ is even}, \ l \text{ is even}, \end{cases}$$

$$w(x_{1,l,1}x_{1,l,2}) = \begin{cases} sa + (s-1)(b-2), & l \text{ is odd,} \\ sa + (s-1)(b-2) + 2, & l \text{ is even,} \end{cases}$$

$$w\left(x_{b-2,j,2}, x_{\frac{a}{2}+1,j+1,1}\right) = \begin{cases} sa + (s-1)(b-2), & l \text{ is odd,} \\ sa + (s-1)(b-2) + 2, & l \text{ is even.} \end{cases}$$

Therefore, there are three different edge weights in the local antimagic labeling of sC_aP_b, which are $sa + (s-1)(b-2)$, $sa + (s-1)(b-2) + 1$, $sa + (s-1)(b-2) + 2$. Thus, we can conclude that $\chi'_{lea}(sC_aP_b) \leq 3$ for $a \equiv 0 \pmod 4$.

Case 2. $a \equiv 2 \pmod 4$.
Define a bijection $f : V(sC_aP_b) \to \{1, 2, 3, \ldots, |V(sC_aP_b)|\}$ to be a local edge antimagic labeling for sC_aP_b as follows.

$$f(x_{k,l,1}) = \begin{cases} sa + (s-1)(b-2) - \left(\frac{a}{2} - k\right) - \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \leq \frac{a}{2}, k \text{ is odd,} \\ \frac{a}{2} - (k-1) + \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right) & k \leq \frac{a}{2}, k \text{ is even,} \\ sa + (s-1)(b-2) + \left(\frac{a}{2} + 1 - k\right) - \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k > \frac{a}{2}, k \text{ is odd,} \\ k - \frac{a}{2} + \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k > \frac{a}{2}, k \text{ is even,} \end{cases}$$

$$f(x_{k,l,2}) = \begin{cases} \frac{a}{2} + \frac{k+1}{2} + \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right) & k \text{ is odd,} \\ sa + (s-1)(b-2) - \left(\frac{a}{2} + \frac{k-2}{2}\right) - \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \text{ is even.} \end{cases}$$

The edge weights are as follows.

$$w(x_{k,l,1}x_{k+1,l,1}) = \begin{cases} sa + (s-1)(b-2), & k < \frac{a}{2}, k \text{ is odd,} \\ sa + (s-1)(b-2) + 2, & k < \frac{a}{2}, k \text{ is even,} \\ sa + (s-1)(b-2) + 1, & k = \frac{a}{2}, \\ sa + (s-1)(b-2), & k > \frac{a}{2}, k \text{ is even,} \\ sa + (s-1)(b-2) + 2, & k > \frac{a}{2}, k \text{ is odd,} \end{cases}$$

$$w(x_{a,l,1}x_{1,l,1}) = sa + (s-1)(b-2) + 1,$$

$$w(x_{k,l,2}x_{k+1,l,2}) = \begin{cases} sa + (s-1)(b-2) + 1, & k \text{ is odd,} \\ sa + (s-1)(b-2) + 2, & k \text{ is even,} \end{cases}$$

$$w\left(x_{1,l,1}x_{1,l,2}\right) = sa + (s-1)(b-2) + 2,$$

$$w\left(x_{b-2,j,2}x_{\frac{a}{2}+1,j+1,1}\right) = sa + (s-1)(b-2) + 2.$$

Therefore, there are three different edge weights in the local antimagic labeling of sC_aP_b, which are $sa + (s-1)(b-2)$, $sa + (s-1)(b-2) + 1$, $sa + (s-1)(b-2) + 2$. Thus, we can conclude that $\chi'_{lea}(sC_aP_b) \leq 3$ for $a \equiv 2 \pmod 4$.

As a result, f induces a proper edge coloring of sC_aP_b using three colors, namely $sa+(s-1)(b-2)$, $sa+(s-1)(b-2)+1$, $sa+(s-1)(b-2)+2$, and we get $\chi'_{lea}(sC_aP_b)\le 3$. Since $\Delta(sC_aP_b)=3$, we obtain $\chi'_{lea}(sC_aP_b)\ge\Delta(sC_aP_b)=3$.

□

In Figures 2 and 3, we give examples of local edge antimagic labeling of $4C_{12}P_4$ and $4C_6P_4$ with $\chi'_{lea}(4C_{12}P_4)=\chi'_{lea}(4C_6P_4)=3$.

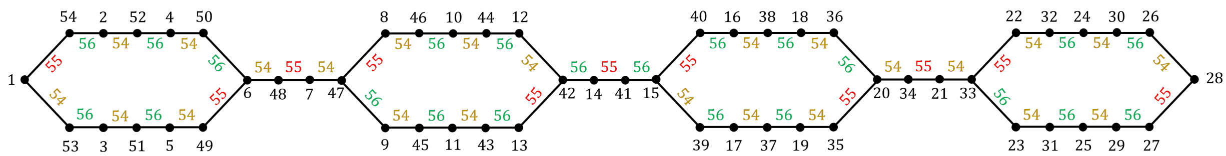

Figure 2: The local edge antimagic labeling of $4C_{12}P_4$ with $\chi'_{lea}(4C_{12}P_4)=3$

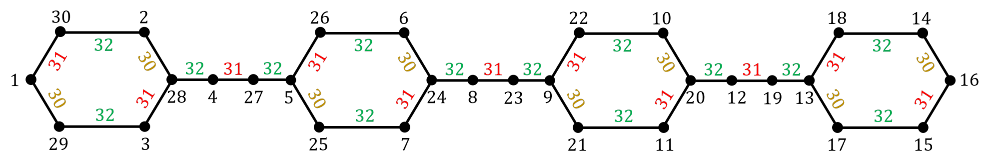

Figure 3: The local edge antimagic labeling of $4C_6P_4$ with $\chi'_{lea}(4C_6P_4)=3$

Theorem 2.2. *Let $b\ge 2$ and $a\ge 4$ be even integers. The local antimagic chromatic index of sP_bC_a is $\chi'_{lea}(sP_bC_a)=3$.*

Proof. The graph sP_bC_a is a connected graph with vertex set $V(sP_bC_a)=\{x_{k,l,1}:1\le k\le b,1\le l\le s\}\cup\{x_{k,l,2}:1\le k\le a-2,1\le l\le s-1\}$ and edge set $E(sP_bC_a)=\{x_{k,l,1}x_{k+1,l,1}:1\le k\le b-1,1\le l\le s\}\cup\{x_{k,l,2}x_{k+1,l,2}:1\le k\le\frac{a}{2}-2,1\le l\le s-1\}\cup\{x_{k,l,2}x_{k+1,l,2}:\frac{a}{2}\le k\le a-3,1\le l\le s-1\}\cup\{x_{1,l,2}x_{1,l+1,1}:1\le l\le s-1\}\cup\{x_{a-2,j,2}x_{1,l+1,1}:1\le l\le s-1\}\cup\{x_{b,j,1}x_{\frac{a-2}{2},j,2}:1\le l\le s-1\}\cup\{x_{b,j,1}x_{\frac{a}{2},j,2}:1\le l\le s-1\}$. Thus, the number of vertices is $|V(sC_aP_b)|=sb+(s-1)(a-2)$ and the number of edges is $|E(sC_aP_b)|=(s-1)a+s(b-1)$.

In Figure 4 we have the graph sP_bC_a and the vertex.

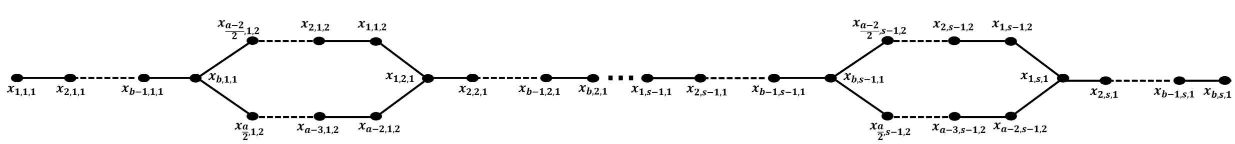

Figure 4: graph sP_bC_a.

Case 1. $a\equiv 0\,(\mathrm{mod}\ 4)$.
Define a bijection $f:V(sP_bC_a)\to\{1,2,3,\ldots,|V(sP_bC_a)|\}$ to be a local edge antimagic labeling for sP_bC_a as follows.

$$f(x_{k,l,1}) = \begin{cases} \frac{k+1}{2} + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, \ l \text{ is odd} \\ sb + (s-1)(a-2) - \frac{k-2}{2} - \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is even}, \ l \text{ is odd} \\ sb + (s-1)(a-2) - \frac{b+a-2}{2} + (2 - \frac{k+1}{2}) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ l \text{ is even}, \\ \frac{b+n+i}{2} + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ l \text{ is even}, \end{cases}$$

$$f(x_{k,l,2}) = \begin{cases} (\frac{a+b-2}{2}) - (k-1) + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, k \leq \frac{a}{2}-1, \ l \text{ is odd}, \\ sb + (s-1)(a-2) - (\frac{a+b-2}{2}) + k + 1 - (a+b-2)\left(\frac{l-1}{2}\right), & k \text{ is even}, \ k \leq \frac{a}{2}-1, \ l \text{ is odd}, \\ sb + (s-1)(a-2) + \left(\frac{a-b+2}{2} - (k+1)\right) - \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is odd}, \ k \geq \frac{a}{2}, \ l \text{ is odd}, \\ (k+3) + \left(\frac{b-2-a}{2}\right) + \left((a+b-2)\left(\frac{l-1}{2}\right)\right), & k \text{ is even}, \ k \geq \frac{a}{2}, \ l \text{ is odd}, \\ sb + (s-1)(a-2) - (a+b-k-3) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ k \leq \frac{a}{2}-1, \ j \text{ is even}, \\ a+b-(k+1) + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ k \leq \frac{a}{2}-1, \ l \text{ is even}, \\ b+k+1 + \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is odd}, \ k \geq \frac{a}{2}, \ l \text{ is even}, \\ sb + (s-1)(a-2) - (b+k-1) - \left((a+b-2)\left(\frac{l-2}{2}\right)\right), & k \text{ is even}, \ k \geq \frac{a}{2}, \ j \text{ is even}, \end{cases}$$

The edge weights are as follows.

$$w(x_{k,l,1}x_{k+1,l,1}) = \begin{cases} sb + (s-1)(a-2) + 1, & k \text{ is odd}, \ l \text{ is odd}, \\ sb + (s-1)(a-2) + 2, & k \text{ is even}, \ l \text{ is odd}, \\ sb + (s-1)(a-2) + 3, & k \text{ is odd}, \ l \text{ is even}, \\ sb + (s-1)(a-2) + 2, & k \text{ is even}, \ l \text{ is even}, \end{cases}$$

$$w(x_{k,l,2}x_{k+1,l,2}) = \begin{cases} sb + (s-1)(a-2) + 3, & k \text{ odd}, \ k < \frac{a}{2}-1, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 1, & k \text{ even}, \ k < \frac{a}{2}-1, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 3, & k \text{ odd}, \ k \geq \frac{a}{2}, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 1, & k \text{ even}, \ k \geq \frac{a}{2}, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 1, & k \text{ odd}, \ k < \frac{a}{2}-1, \ l \text{ even}, \\ sb + (s-1)(a-2) + 3 & k \text{ even}, \ k < \frac{a}{2}-1, \ l \text{ even}, \\ sb + (s-1)(a-2) + 1, & k \text{ odd}, \ k \geq \frac{a}{2}, \ l \text{ even}, \\ sb + (s-1)(a-2) + 3, & k \text{ even}, \ k \geq \frac{a}{2}, \ l \text{ even}, \end{cases}$$

$$w(x_{b,j,1}x_{\frac{a}{2},j,2}) = \begin{cases} sb + (s-1)(a-2) + 3, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 1, \ l \text{ even}, \end{cases}$$

$$w(x_{b,j,1}x_{\frac{a}{2}-1,j,2}) = sb + (s-1)(a-2) + 2,$$

$$w(x_{1,l,2}x_{1,l+1,1}) = \begin{cases} sb + (s-1)(a-2) + 1, \ l \text{ odd}, \\ sb + (s-1)(a-2) + 3, \ l \text{ even}, \end{cases}$$

$$w(x_{a-2,j,2}x_{1,l+1,1}) = sb + (s-1)(a-2) + 2,$$

Therefore, there are three different edge weights in the local antimagic labeling of sP_bC_a, which are $sb+(s-1)(a-2)+1$, $sb+(s-1)(a-2)+2$, $sb+(s-1)(a-2)+2$. Thus, we can conclude that $\chi'_{lea}(sP_bC_a) \leq 3$ for $a \equiv 0 \pmod 4$.

Case 2. $a \equiv 2 \pmod 4$.
Define a bijection $f : V(sP_bC_a) \to \{1, 2, 3, \ldots, |V(sP_bC_a)|\}$ to be a local edge antimagic labeling for sP_bC_a as follows.

$$f(x_{k,l,1}) = \begin{cases} \frac{k+2}{2} + \left(\left(\frac{a+b-2}{2}\right)(l-1)\right), & k \text{ is odd,} \\ sb + (s-1)(a-2) - \frac{k-2}{2} - \left(\left(\frac{a+b-2}{2}\right)(l-1)\right), & k \text{ is even,} \end{cases}$$

$$f(x_{k,l,2}) = \begin{cases} sb + (s-1)(a-2) - \left(\frac{a+b-2}{2} - (k+1)\right) - \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \text{ is odd, } k \le \frac{a}{2} - 1, \\ \frac{a+b}{2} - (k) + \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \text{ is even, } k \le \frac{a}{2} - 1, \\ k + 3 + \left(\frac{b-2-a}{2}\right) + \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \text{ is odd, } k \ge \frac{a}{2}, \\ sb + (s-1)(a-2) + \left(\frac{a-b+2}{2} - k - 1\right) - \left(\left(\frac{a+b}{2} - 1\right)(l-1)\right), & k \text{ is even, } k \ge \frac{a}{2}. \end{cases}$$

The edge weights are as follows.

$$w(x_{k,l,1}x_{k+1,l,1}) = \begin{cases} sb + (s-1)(a-2) + 1, & k \text{ is odd,} \\ sb + (s-1)(a-2) + 2, & k \text{ is even,} \end{cases}$$

$$w(x_{k,l,2}x_{k+1,l,2}) = \begin{cases} sb + (s-1)(a-2) + 1, & k \text{ odd, } k < \frac{a}{2} - 1, \\ sb + (s-1)(a-2) + 3 & k \text{ even, } k < \frac{a}{2} - 1, \\ sb + (s-1)(a-2) + 1, & k \text{ odd, } k \ge \frac{a}{2}, \\ sb + (s-1)(a-2) + 3, & k \text{ even, } k \ge \frac{a}{2}, \end{cases}$$

$$w(x_{b,j,1}x_{\frac{a}{2},j,2}) = sb + (s-1)(a-2) + 3,$$

$$w(x_{b,j,1}x_{\frac{a}{2}-1,j,2}) = sb + (s-1)(a-2) + 2,$$

$$w(x_{1,l,2}x_{1,l+1,1}) = sb + (s-1)(a-2) + 3,$$

$$w(x_{a-2,j,2}x_{1,l+1,1}) = sb + (s-1)(a-2) + 2,$$

Therefore, there are three different edge weights in the local antimagic labeling of sP_bC_a, which are $sb + (s-1)(a-2) + 1$, $sb + (s-1)(a-2) + 2$, $sb + (s-1)(a-2) + 2$. Thus, we can conclude that $\chi'_{lea}(sP_bC_a) \le 3$ for $a \equiv 2 \pmod 4$.

As a result, f induces a proper edge coloring of sP_bC_a using three colors, namely $sb + (s-1)(a-2) + 1$, $sb + (s-1)(a-2) + 2$, $sb + (s-1)(a-2) + 2$, and we get $\chi'_{lea}(sP_bC_a) \le 3$. Since $\Delta(sP_bC_a) = 3$, we obtain $\chi'_{lea}(sP_bC_a) \ge \Delta(sP_bC_a) = 3$. □

In Figures 5 and 6, we give the examples of local edge antimagic labeling of $4P_4C_{12}$ and $5P_4C_6$ with $\chi'_{lea}(4P_4C_{12}) = \chi'_{lea}(5P_4C_6) = 3$.

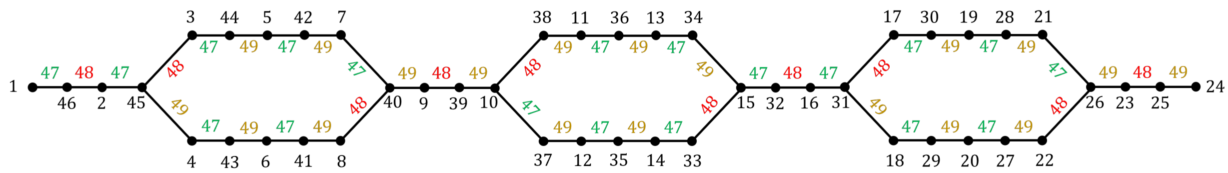

Figure 5: The local edge antimagic labeling of $4P_4C_{12}$ with $\chi'_{lea}(4P_4C_{12}) = 3$

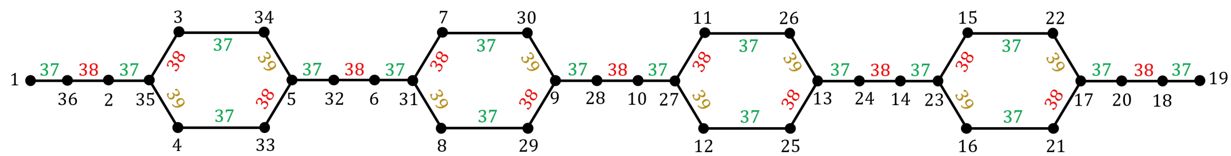

Figure 6: The local edge antimagic labeling of $5P_4C_6$ with $\chi'_{lea}(5P_4C_6) = 3$

Open Problem

Determine $\chi'_{lea}(sC_aP_b)$ and $\chi'_{lea}(sP_bC_a)$ for a or b are odd.

Acknowledgement

Part of this research is funded by Hibah Publikasi Terindeks Internasional (PUTI) Q2 number NKB-740/UN2.RST/HKP.05.00/2023 dan Hibah Publikasi Pascasarjana FMIPA UI number PKS-040/UN2.F3.D/PPM.00.02/2024.

References

[1] I. H. Agustin, M. Hasan, Dafik, R. Alfarisi, and R. M. Prihandini, Local edge antimagic coloring of graphs, *Far East Journal of Mathematical Sciences*, **102** (2017), 1925–1941.

[2] S. Arumugam, K. Premalatha, M. Bacă, and A. Semaničová-Feňovčíková, Local antimagic vertex coloring of a graph, *Graphs and Combinatorics*, **33** (2017), 275–285.

[3] J. Bensmail, F. Desormeaux, and S. Senhaji, Local antimagic edge labeling of trees, *Journal of Graph Theory*, **91**(4) (2019), 379–394.

[4] H. Choi, S. Kim, and J. Park, Local antimagic edge colorings in generalized graph families, *Discrete Applied Mathematics.*, **278** (2020), 16–26.

[5] R. Diestel, *Graph Theory*, 5th ed., Berlin: Springer, 2017.

[6] N. Hartsfield and G. Ringel, *Pearls in Graph Theory*, Academic Press, 1990.

[7] S. Lee, J. Liu, and X. Zhang, Local antimagic edge labeling for planar graphs, *Journal of Combinatorics*, **11**(2) (2020), 199–217.