Indonesian Journal of Combinatorics **2** (1) (2018), 1–13

INDONESIAN JOURNAL OF COMBINATORICS

3-difference cordial labeling of some path related graphs

R. Ponraj[a], M. Maria Adaickalam[b], R. Kala[c]

[a]*Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India.*
[b]*Research Scholar, Manonmaniam Sundaranar University, Tirunelveli-627012, India.*
[c]*Manonmaniam Sundaranar University, Tirunelveli-627012, India.*

ponrajmaths@gmail.com, mariaadaickalam@gmail.com, karthipyi91@yahoo.co.in.

Abstract

Let G be a (p, q)-graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map where k is an integer, $2 \leq k \leq p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of triangular snake, alternate triangular snake, alternate quadrilateral snake, irregular triangular snake, irregular quadrilateral snake, double triangular snake, double quadrilateral snake, double alternate triangular snake, and double alternate quadrilateral snake.

Keywords: Cycle, path, triangular snake, quadrilateral snake, difference cordial.
Mathematics Subject Classification: 05C78
DOI: 10.19184/ijc.2018.2.1.1

1. Introduction

Graphs considered in this paper are finite and simple. Graph labeling is used in several areas of science and technology such as coding theory, astronomy, circuit design etc. For more details on application of graph labeling, see [2]. Let G be a (p, q)-graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices

Received: 30 Aug 2017, Revised: 23 Dec 2017, Accepted: 07 Feb 2018.

labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1.

A graph with a k-difference cordial labeling is called a k-difference cordial graph. The variations of cordial labeling concept was introduced in [1]. R. Hasni et al.[4] investigate the product cordial and total product cordial labeling behavior of some graphs. The notion of difference cordial labeling was introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [5]. Seoud and Salman [11], studied the difference cordial labeling of some families of graphs, namely, ladder, triangular ladder, grid, step ladder and two sided step ladder graphs etc. Recently, Ponraj et al. [6] introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling of star, m copies of star etc. In [7, 8, 9, 10], they discussed the 3-difference cordial labeling of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, double wheel, union of graphs with the star, union of graphs with splitting graph of star, union of graphs with subdivided star, union of graphs with bistar, $P_n \cup P_n$, $(C_n \odot K_1) \cup (C_n \odot K_1)$, $F_n \cup F_n$, $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, mC_4, $spl(K_{1,n})$, $DS(B_{n,n})$, $C_n \odot K_2$, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$, $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$ and some more graphs. In this paper we investigate 3-difference cordial labeling of triangular snake, alternate triangular snake, alternate quadrilateral snake, irregular triangular snake, irregular quadrilateral snake, double triangular snake, double quadrilateral snake, double alternate triangular snake, and double alternate quadrilateral snake. Terms are not defined here follows from Harary [3].

2. 3-Difference cordial labeling

We investigate the 3-difference cordial labeling of some path related graphs. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Theorem 2.1. *If $n \equiv 0, 1, 2 \pmod 4$ then the triangular snake T_n is a 3-difference cordial graph.*

Proof. Let P_n be a path $u_1u_2 \dots u_n$. Let $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) \cup \{u_iv_i, v_iu_{i+1} : 1 \leq i \leq n-1\}$. In this graph T_n, $|V(T_n)| = 2n-1$ and $|E(T_n)| = 3n-3$.
Case 1. $n \equiv 0 \pmod 4$.
Considering the vertices v_i, assign the label 2 to the vertices $v_1, v_5, v_9, ...$ Next assign the label 3 to the vertices $v_2, v_6, v_{10}, ...$ Then we assign the label 1 to the vertices $v_3, v_7, v_{11}, ...$ Now we consider the vertices v_{4i}. Assign the label 1 to the vertices v_{4i} for the values i = 2, 5, 8,... Next we assign the label 2 to the vertices v_{4i} for i = 3, 6, 9,... Then for the values i = 1, 4, 7,... assign the label 3 to the vertices u_{4i}. Next we move to the vertices u_i. Now We consider the vertices u_i. Assign the label 1 on the vertices $u_2, u_6, u_{10}, ...$ Next assign the label 3 to the vertices $u_3, u_7, u_{11}, ...$ Then assign the label 2 to the vertices $u_4, u_8, u_{12}, ...$ Next we consider the vertices u_{4i+1}, assign the label 1 to the vertices u_{4i+1} for the values i = 0, 3, 6,... Then assign the label 2 to the vertices u_{4i+1} for i = 1, 4, 7,... Then for the values i = 2, 5, 8,...assign the label 3 to the vertices u_{4i+1}.
Case 2. $n \equiv 1 \pmod 4$.
Consider the vertices v_i. Assign the label 2 to the vertices $v_2, v_6, v_{10}, ...$ Next assign the label 3 to the vertices $v_3, v_7, v_{11}, ...$ Then assign the label 1 to the vertices $v_4, v_8, v_{12}, ...$ Now we consider the vertices v_{4i+1}. Assign the label 1 to the vertices v_{4i+1} for the values i = 0, 3, 6,... Then assign the

label 2 to the vertices v_{4i+1} for $i = 1$, 4, 7,... Then for the values $i = 2$, 5, 8,... assign the label 3 to the vertices v_{4i+1}. Next we move to the path vertices u_i. First we fix the label 2 to the vertex u_1. Now we consider the vertices u_i as in the label 1 to the vertices $u_3, u_7, u_{11}, ...$ Next we assign the label 3 to the vertices $u_4, u_8, u_{12}, ...$ Then we assign the label 2 to the vertices $u_5, u_9, u_{13}, ...$ Now we consider the vertices u_{4i+2}. Assign the label 3 to the vertices u_{4i+2} for the values $i = 0$, 3, 6,... Then assign the label 1 to the vertices u_{4i+2} for $i = 1$, 4, 7,... Then for the values $i = 2$, 5, 8,... assign the label 2 to the vertices u_{4i+2}.

Case 3. $n \equiv 2 \pmod 4$.

First we fix the label 1 to the vertex v_1. Now we consider the vertices v_i as in the label 2 to the vertices $v_3, v_7, v_{11}, ...$ Next assign the label 3 to the vertices $v_4, v_8, v_{12}, ...$ then assign the label 1 to the vertices $v_5, v_9, v_{13}, ...$ We now consider the vertices v_{4i+2}. Assign the label 1 to the vertices v_{4i+2} for the values $i = 0$, 3, 6,... Next we assign the label 2 to the vertices v_{4i+2} for $i = 1$, 4, 7,... Then for the values $i = 2$, 5, 8,... assign the label 3 to the vertices v_{4i+2}. Next we move to the path vertices u_i. Now we fix the labels 2 and 3 to the vertices u_1 and u_2 respectively. Next we consider the vertices u_i. Assign the label 1 to the vertices $u_4, u_8, u_{12}, ...$ Next assign the label 3 to the vertices $u_5, u_9, u_{13}, ...$ Then assign the label 2 to the vertices $u_6, u_{10}, u_{14}, ...$ Next we consider the vertices u_{4i+3}. Assign the label 2 to the vertices u_{4i+3} for the values $i = 0$, 3, 6,... Next assign the label 3 to the vertices u_{4i+3} for $i = 1$, 4, 7,... Then for the values $i = 2$, 5, 8,... assign the label 1 to the vertices u_{4i+3}. The vertex and edge condition are given in table 1 and 2 respectively.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 2, 5, 8 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$
$n \equiv 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 0 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$
$n \equiv 6, 9 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$
$n \equiv 1, 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$

Table 1. Vertex label

values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{3n-2}{2}$	$\frac{3n-4}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n-3}{2}$	$\frac{3n-3}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n-4}{2}$	$\frac{3n-2}{2}$

Table 2. Edge labels

□

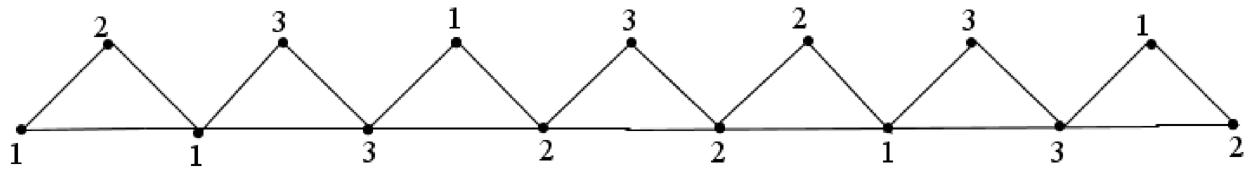

Figure 1. A 3-difference cordial labeling for T_8

Next is the alternate triangular snake. An alternate triangular snake AT_n is obtained from a path $u_1u_2...u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Theorem 2.2. *Alternate triangular snakes are 3-difference cordial graphs.*

Proof. **Case 1.** Let the first triangle starts from u_2 and the last triangle be ends with u_{n-1}.
In this case $|V(AT_n)| = \frac{3n-2}{2}$ and $|E(AT_n)| = 2n-3$. Assign label 2 to the vertices $v_1, v_2, ...$ We assign the label to the path vertices $u_1, u_2, ...u_n$ in the pattern 1, 3, 1, 3,.... 1, 3. Note that in this process the vertex u_n and u_{n-1} received the labels 1, 3 respectively. In this case the vertex condition is given by $v_f(1) = v_f(3) = \frac{n}{2}$ and $v_f(2) = \frac{n}{2} - 1$. Also the edge condition is $e_f(0) = n-1$ and $e_f(1) = n-2$.
Case 2. Let the first triangle starts from u_1 and the last triangle be ends with u_n.
Here $|V(AT_n)| = \frac{3n}{2}$ and $|E(AT_n)| = 2n-1$. Assign the label to the vertices as in case 1. The vertex and edge conditions are given by $v_f(1) = v_f(2) = v_f(3) = \frac{n}{2}$ and $e_f(0) = n-1$ and $e_f(1) = n$ respectively.
Case 3. Let the first triangle starts from u_2 and the last triangle be ends with u_n.
Note that in this case $|V(AT_n)| = \frac{3n-1}{2}$ and $|E(AT_n)| = 2n-2$. Assign the label to the vertices as in case 1. It is easy to verify that the last vertex u_n received the label 1 in this case. This vertex labeling is a 3-difference cordial labeling follows from the vertex and edge condition $v_f(1) = \frac{n+1}{2}$, $v_f(2) = v_f(3) = \frac{n-1}{2}$ and $e_f(0) = e_f(1) = n-1$.
Case 4. Let the first triangle starts from u_1 and the last triangle be ends with u_{n-1}.
This case is equivalent to case 3. □

Now we look into alternate quadrilateral snake. An alternate quadrilateral snake AQ_n is obtained from a path $u_1u_2...u_n$ by joining u_i,u_{i+1} (alternatively) to new vertices v_i, w_i respectively and then joining v_i and wi. That is every alternate edge of a path is replaced by a cycle C_4.

Theorem 2.3. *All alternate quadrilateral snakes are 3-difference cordial graphs.*

Proof. **Case 1.** Let the first C_4 be starts from u_2 and the last C_4 be ends with u_{n-1}.
Note that in this case $|V(AQ_n)| = 2n-2$ and $|E(AQ_n)| = \frac{5n-8}{2}$. We consider the vertices in the path. Assign the labels 2, 1, 1, 3 to the vertices u_1, u_2, u_3, u_4. Next we assign the labels 2, 1, 1, 3 to the next four vertices u_5, u_6, u_7, u_8. Continuing this way, assign the label to the next four vertices and so on. Clearly in this process the vertex u_n received the labels 1 or 3 according as $n \equiv 2 \pmod 4$ or $n \equiv 0 \pmod 4$. Then we move to the vertices v_i and w_i. Fix the labels 2 and 3 to the vertices v_1 and w_1 respectively. Assign the labels 1 and 3 to the vertices v_{12i+2} and w_{12i+2}

for the values $i = 0, 1, 2, 3,...$ Then assign the label 2 to the vertices v_{12i+3} and w_{12i+3} for $i = 0, 1, 2, 3,...$ For the values of $i = 0, 1, 2, 3,...$ assign the label 3 to the vertices v_{12i+4} and w_{12i+4}. Next assign the labels 2 and 3 to the vertices v_{6i+5} and w_{6i+5} for all the vertices of $i = 0, 1, 2, 3...$ Then we assign the labels 1 and 3 to the vertices v_{6i} and w_{6i} for $i = 1, 2, 3,...$ For all the values of $i = 1, 2, 3,...$ assign the label 2 to the vertices v_{6i+1} and w_{6i+1} respectively. Now we assign the label 3 to the vertices v_{12i+8} and w_{12i+8} for all the values of $i = 0, 1, 2, 3,...$ Then assign the labels 2 and 1 to the vertices v_{12i+9} and w_{12i+9} for $i = 0, 1, 2, 3,...$ For the values $i = 0, 1, 2, 3,...$ assign the labels 3 and 2 to the vertices v_{12i+10} and w_{12i+10} respectively. Clearly Clearly the vertex and edge label satisfy the condition of this case are given in table 3 and table 4.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 4, 10, 16, 22 \pmod{24}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 6 \pmod{24}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
$n \equiv 8 \pmod{24}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-4}{3}$
$n \equiv 0, 12, 18 \pmod{24}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$

Table 3. Vertex label

values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod{4}$	$\frac{5n-8}{4}$	$\frac{5n-8}{4}$
$n \equiv 2 \pmod{4}$	$\frac{5n-10}{4}$	$\frac{5n-6}{4}$

Table 4. Edge label

Case 2. Let the first C_4 be starts from u_1 and the last C_4 be ends with u_n.
Here $|V(A(Q_n))| = 2n$ and $|E(A(Q_n))| = \frac{5n-2}{2}$. First we consider the vertices of a path. Assign the labels 1, 3, 2, 1 to the first four vertices u_1, u_2, u_3, u_4 respectively. Next we assign the labels 1, 3, 2, 1 to the next four vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this, assign the next four vertices and so on. Clearly the vertex u_n received the label 1 or 3 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{4}$. Consider the vertices v_{6i+2} and v_{6i+4}. Now we assign the label 3 to the vertices v_{6i+2} and v_{6i+4} for all the values of $i = 0, 1, 2, 3,...$ Assign the label 3 to the vertices w_{2i} for $i = 1, 2, 3,...$ Next we assign the label 2 to the vertice v_{6i+3} and v_{6i+} for all the values of $i = 0, 1, 2, 3,...$ For all the vvalues of $i = 1, 2, 3,...$ assign the label 2 to the vertices w_{2i+1}. Now we consider the vertices v_{6i} and v_{6i+1} for all the vaalues $i = 1, 2, 3,...$ The vertex and edge condition of this case are given in table 5 and tablet6.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 6 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$
$n \equiv 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod{12}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

Table 5. Vertex label

values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{5n-4}{4}$	$\frac{5n}{4}$
$n \equiv 2 \pmod 4$	$\frac{5n-2}{4}$	$\frac{5n-2}{4}$

Table 6. Edge label

Case 3. Let the first C_4 be starts from u_2 and the last C_4 be ends with u_n.
Note that $|V(A(Q_n))| = 2n - 1$ and $|E(A(Q_n))| = \frac{5n-5}{2}$. First we fix the label 1 to the vertex u_1. Then we consider the path vertices u_i as in the labels 1, 3, 2, 1 to the four vvertices u_2, u_3, u_4, u_5 respectively. Next assign the labels 1, 3, 2, 1 to the net four vertices u_6, u_7, u_8, u_9 respectively. Continuing like this to assign the label to the next four vertices and so on. In this process, the last vertex u_n received the label 3 or 1 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Consider the vertices v_{6i+1} and v_{6i+3}. Assign the label 2 to the vertices v_{6i+1} and v_{6i+3} for the values $i = 0$, 1, 2, 3,... Next assign the label 3 to the vertices v_{6i+2} and v_{6i+4} for $i = 0$, 1, 2,... For the values of $i = 0$, 1, 2,.... assign the label 1 to the vertices v_{6i+5}. Then assign the label 1 to the vertices v_{6i} for $i = 1$, 2, 3,... Now we move to the vertices w_i. Assign the label 2 to the vertices w_{2i+1} for all the values of $i = 0$, 1, 2, 3,... Then assign the label 3 to the vertices w_{2i} for $i = 1$, 2, 3,... Clearly the vertex and edge condition of this case are given in table 7 and table 8.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 5, 11 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$
$n \equiv 3 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$
$n \equiv 7 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 9 \pmod{12}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$

Table 7. Vertex label

values of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 4$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$
$n \equiv 3 \pmod 4$	$\frac{5n-3}{4}$	$\frac{5n-7}{4}$

Table 8. Edge label

Case 4. Let the first C_4 be starts from u_1 and the last C_4 be ends with u_{n-1}.
This case is equivalent to case 3. □

Next investigation is about the irregular triangular snakes. The irregular triangular snake IT_n is obtained from the path $u_1 u_2 ... u_n$ with vertex set $V(IT_n) = V(P_n) \cup \{v_i : 1 \le i \le n-2\}$ and the edge set $E(IT_n) = E(P_n) \cup \{u_i v_i, v_i u_{i+2} : 1 \le i \le n-2\}$.

Theorem 2.4. *The irregular triangular snake is a 3-difference cordial graph.*

Proof. Clearly $|V(IT_n)| = 2n - 2$ and $|E(IT_n)| = 3n - 5$. Let $n = 3t + r$, $0 \le r < 8$. Assign the labels 1, 3, 2, 2, 3, 1, 2, 2 to the first eight path vertices. Next eight path vertices are labeled by 1, 3, 2, 2, 3, 1, 2, 2. Proceeding like this, assign the label to the 8t vertices of the path. Note that the vertex u_{8t} receive the label 2. Assign the label to the next r vertices $u_{8t+1}, u_{8t+2}, ..., u_{8t+r}$ by the sequence of integers 1, 3, 2, 2, 3, 1,.... Next assign the label to the first five vertices v_1, v_2, v_3, v_4, v_5 by the integers 1, 3, 2, 1, 3. Then assign the labels 1, 3, 2, 1, 3 to the next five vertices $v_6, v_7, v_8, v_9, v_{10}$. Continuing this way, assign the label to the next five vertices and so on. If all the vertices v_i are labeled, then stop. Otherwise there are some non labeled vertices and the number of labeled verties is less than or equal to 4. Assign the labels 1, 3, 2, 1 to the non labeled vertices. That is if only one non labeled vertices is exists we use the label 1 only if it is two then we use the labels 1, 3 and so on. This labeling is clearly 3-difference cordial labeling follows from table 9.

Value of n	$e_f(0)$	$e_f(1)$
n is odd	$\frac{3n-5}{2}$	$\frac{3n-5}{2}$
n is even	$\frac{3n-4}{2}$	$\frac{3n-6}{2}$

Table 9. Edge label

□

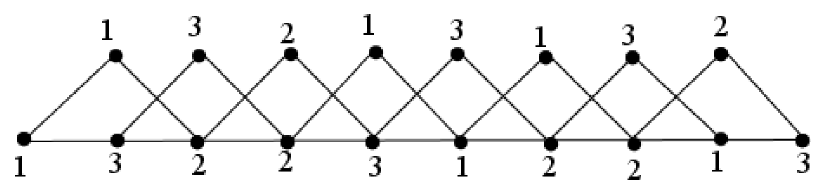

Figure 2. A 3-difference cordial labeling of irregular triangular snake IT_{10}

The irregular quadrilateral snake IQ_n is obtained from the path $P_n : u_1u_2...u_n$ with vertex set $V(IQ_n) = V(P_n) \cup \{v_i, w_i : 1 \leq i \leq n-2\}$ and the edge set $E(IQ_n) = E(P_n) \cup \{u_iv_i, w_iu_{i+2} : 1 \leq i \leq n-2\}$.

Theorem 2.5. *The irregular quadrilateral snake is a 3-difference cordial graph.*

Proof. Clearly $|V(IQ_n)| = 3n-4$ and $|E(IQ_n)| = 4n-7$ respectively. We consider the path vertices u_i. Assign the labels 1, 2, 3 to the first three path vertices u_1, u_2, u_3 respectively. Then assign the label 1 to the remaining path vertices $u_4, u_5, u_6, ...$ Next assign the label 2 to the vertices v_i and assign the label 3 to the vertices w_i. Clearly $e_f(0) = 2n-4$and $e_f(1) = n-2$ and $v_f(1) = n-1$ and $v_f(2) = v_f(3) = n-1$. Therefore IQ_n is 3-difference cordial. □

A double triangular snake DT_n consists of two triangular snakes that have a common path. That is a double triangular snake is obtained from a path $u_1u_2...u_n$ by joining u_i and u_{i+1} to a new vertex $v_i (1 \leq i \leq n-1)$ and to a new vertex w_i $(1 \leq i \leq n-1)$.

Theorem 2.6. *Double triangular snake DT_n is a 3-difference cordial graph.*

Proof. Here $|V(DT_n)| = 3n-2$ and $|E(DT_n)| = 5n-5$. First we consider the path vertices u_i. Assign the labels 1, 3 to the vertices u_1, u_2 respectively. Then assign the labels 2, 3, 3, 3 to the path vertices u_3, u_4, u_5, u_6. Using the same pattern assign the labels 2, 3, 3, 3 to the next four path vertices u_7, u_8, u_9, u_{10}. Continuing this way assign the next four vertices and so on. If all the path vertices are labeled in this way then we stop the process. Otherwise there are some non labeled vertices exists and in the case the number of non labeled vertices less than or equal to 3. Assign the labels 2, 3, 3 to the non labeled vertices. If only one non labeled vertex exists then we use the label 2 only. If there are two labeled vertices then we use the labels 2, 3. Next we move to the vertices v_i, w_i. Assign the label 1 to all the vertices v_i $(1 \leq i \leq n-1)$. Assign the label 2 to the vertex w_1. Then assign the labels 3, 2, 2, 2 to the four vertices w_2, w_3, w_4, w_5. Next we assign the labels 3, 2, 2, 2 to the next four vertices w_6, w_7, w_8, w_9. Proceeding like this, if all the vertices w_i are labeled then stop. Otherwise next non labeled vertices in the sequence 3, 2, 2. This labeling is a 3-difference cordial labeling and its vertex and edge condition is given by $v_f(1) = n$ and $v_f(2) = v_f(3) = n-1$.

Value of n	$e_f(0)$	$e_f(1)$
n is odd	$\frac{5n-5}{2}$	$\frac{5n-5}{2}$
n is even	$\frac{5n-4}{2}$	$\frac{5n-6}{2}$

Table 10. Edge label

□

A double quadrilateral snake DQ_n consists of two quadrilateral snake have a common path.

Theorem 2.7. *The double quadrilateral snake is a 3-difference cordial graph.*

Proof. Let $V(DQ_n) = \{u_i : 1 \leq i \leq n\} \cup \{v_i, w_i, x_i, y_i : 1 \leq i \leq n-1\}$ and $E(DQ_n) = \{u_iu_{i+1}, u_iv_i, v_iw_i, u_ix_i, x_iy_i, w_iu_{i+1}, y_iu_{i+1} : 1 \leq i \leq n-1\}$. Clearly $|V(DQ_n)| = 5n-4$ and $|E(DQ_n)| = 7n-7$. Consider the path vertices u_i. Assign the labels 1, 1, 2, 2 to the first four vertices u_1, u_2, u_3, u_4 respectively. Then assign the labels 1, 1, 2, 2 to the next four vertices u_5, u_6, u_7, u_8. Proceeding like this, assign the next four vertices and so on. If all the path vertices are labeled in this way then the labelling is complete. Otherwise there are some non labeled vertices exist. If the number of non labeled vertices less than or equal to 3, then assign the labels 1, 1, 2 to the non labeled vertices. If only one non labeled vertex exists then we use the label 1 only. If there are two unlabelled vertices, then we use the labels 1, 1. Now we move to the vertices v_i and w_i. Assign the labels 2, 1, 3, 2 to the vertices v_1, v_2, v_3, v_4 respectively. Now we assign the labels 3, 1, 2, 1 to the vertices w_1, w_2, w_3, w_4 respectively. Next assign the label 2 to the vertices $v_{12i+5}, v_{12i+7}, v_{12i+9}, v_{12i+11}$ for all the values of $i = 0, 1, 2, 3,\ldots.$ Then assign the label 1 to the vertices $w_{12i+5}, w_{12i+6}, w_{12i+9}, w_{12i+10}$ for $i = 0, 1, 2, 3,\ldots.$ Now for all the values of $i = 1, 2, 3,\ldots$ assign the label 2 to the vertices $v_{12i+1}, v_{12i+3}, v_{12i+4}$. Next we assign the label 1 to the vertices $w_{12i+1}, w_{12i+2}, w_{12i+4}$ for all the values $i = 0, 1, 2, 3,\ldots.$ Then we assign the label 1 to the vertices v_{12i+6}, v_{12i+10} for all the values of $i = 0, 1, 2, 3,\ldots.$ For all the values of $i = 0, 1, 2, 3,\ldots.$ assign the label 3 to the vertices $w_{12i+7}, w_{12i+8}, w_{12i+11}$. Now assign the label 1 to the vertices v_{12i}, v_{12i+2} for all the values of $i = 0, 1, 2, 3,\ldots.$ Next assign the label 3 to the vertices w_{12i}, w_{12i+3} for all the values of $i = 0, 1, 2, 3,....$ We consider the vertices x_i and y_i. Assign the labels 2, 3, 1, 2 to the vertices x_1, x_2, x_3, x_4 respectively. Then we assign the labels 2, 3, 1, 2 to the next four vertices x_5, x_6, x_7, x_8 respectively. Continuing this way we assign the next four vertices and so on. If all the vertices are labeled in this way, then stop. Otherwise there are some non labeled vertices exist. If the number of non labeled vertices less than or equal to 3, then assign the labels 2, 3, 1 to the non labeled vertices. If only one unlabeled vertex exists, then we use the label 2 only. If there are two then we use the labels 2, 3. Now we assign the label 3 to all the vertices y_i $(1 \leq i \leq n-1)$. This labeling is 3-difference cordial labeling follows from the following tables.

values of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 2$	$\frac{7n-7}{2}$	$\frac{7n-7}{2}$
$n \equiv 0 \pmod 2$	$\frac{7n-8}{2}$	$\frac{7n-6}{2}$

Table 11. Edge label

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 2, 5, 8, 11 \pmod{12}$	$\frac{5n-4}{3}$	$\frac{5n-4}{3}$	$\frac{5n-4}{3}$
$n \equiv 3 \pmod{12}$	$\frac{5n-3}{3}$	$\frac{5n-6}{3}$	$\frac{5n-3}{3}$
$n \equiv 7 \pmod{12}$	$\frac{5n-2}{3}$	$\frac{5n-5}{3}$	$\frac{5n-5}{3}$
$n \equiv 6 \pmod{12}$	$\frac{5n-3}{3}$	$\frac{5n-3}{3}$	$\frac{5n-6}{3}$
$n \equiv 0, 9 \pmod{12}$	$\frac{5n-6}{3}$	$\frac{5n-3}{3}$	$\frac{5n-3}{3}$
$n \equiv 10 \pmod{12}$	$\frac{5n-5}{3}$	$\frac{5n-2}{3}$	$\frac{5n-5}{3}$
$n \equiv 1, 4 \pmod{12}$	$\frac{5n-5}{3}$	$\frac{5n-5}{3}$	$\frac{5n-2}{3}$

Table 12. Vertex label

□

A double alternate triangular snake DAT_n consists of two alternate triangular snakes that have a common path. That is a double alternate triangular snake is obtained from a path $u_1u_2....u_n$ by joining u_i and u_{i+1} (alternatively) to two vertices v_i and w_i.

Theorem 2.8. *Double alternate triangular snake DAT_n is a 3-difference cordial graph.*

Proof. **Case 1.** The triangle starts from u_1 and end with u_n.
In this case $|V(DAT_n)| = 2n$ and $|E(DAT_n)| = 3n-1$. Assign the label 1 to the path vertex u_1. Now we assign the label 2 to the vertices $u_{12i+2}, u_{12i+3}, u_{12i+6}, u_{12i+7}, u_{12i+10}, u_{12i+11}$ for all the values of $i = 0, 1, 2, 3,...$ Then we assign the labels 3 to the vertices u_{12i+4}, u_{12i+5}, u_{12i+8}, u_{12i+9} for all the values of $i = 0, 1, 2, 3,...$ For all the values of $i = 0, 1, 2, 3,...$ assign the label 1 to the vertices u_{12i} and u_{12i+1}. Next we move to the vertices v_i and w_i. Assign the labels 2, 1, 3, 1, 2, 3 to the first six vertices $v_1, v_2, v_3, v_4, v_5, v_6$ respectively. Then we assign the labels 2, 1, 3, 1, 2, 3 to the next six vertices $v_7, v_8, v_9, v_{10}, v_{11}, v_{12}$ respectively. Proceeding like this assign the label to the next six vertices and so on. If all the vertices v_i are labeled then stop the process. Otherwise there are some non labeled vertices and number of labeled vertices is less than or equal to 5. Now assign the

labels 2, 1, 3, 1, 2 to the non labeled vertices. If four non labeled vertices are exist then assign the labels 2, 1, 3, 1 to th non labeled vertices. If the number of non labeled vertices is 3 then assign the labels 2, 1, 3 to the non labeled vertices. If only one non labeled vertex exist then assign the label 2 only. If it is two then assign the labels 2, 1 to the non labeled vertices. Consider the vertices w_i. Assign the labels 3, 1, 1 to the first three vertices w_1, w_2, w_3 respectively. Then we assign the labels 3, 1, 1 to the next three vertices w_4, w_5, w_6 respectively. Continuing this way we assign the label to the next three vertices and so on. If all the vertices w_i are labeled, then stop the process. Otherwise there are some non labeled vertices exists. If the number of non labeled vertices are less than or equal to 2 then assign the labels 3, 1 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 3 only. The edge condition is given by $e_f(0) = \frac{3n-2}{2}$ and $e_f(1) = \frac{3n}{2}$. Also the vertex condition is given by a table 13.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 6 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 4 \pmod{12}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$
$n \equiv 8 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$
$n \equiv 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n-1}{3}$

Table 13. Vertex label

Case 2. The triangle starts from u_2 and end with u_{n-1}.
In this case $|V(DAT_n)| = 2n - 2$ and $|E(DAT_n)| = 3n - 5$. First we consider the path vertices u_i as in the labels 1, 3, 2, 2 to the first four path vertices u_1, u_2, u_3, u_4 respectively. Then we assign the labels 1, 3, 2, 2 to the next four path vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this assign the label to the next four vertices and so on. Clearly in this process the vertex u_n received the label 2 or 3 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{4}$. Next we move to the vertices v_i. Assign the labels 1, 2, 3, 2, 3, 1 to the six vertices $v_1, v_2, v_3, v_4, v_5, v_6$ respectively. Then we assign the labels 1, 2, 3, 2, 3, 1 to the next six vertices $v_7, v_8, v_9, v_{10}, v_{11}, v_{12}$ respectively. Continuing this process assign the label to the next six vertices and so on. If all the vertices of v_i are labeled then we stop the process. Otherwise there are some non labeled vertices are exist. If the number of non labeled vertices are less than or equal to 5 then assign the labels 1, 2, 3, 2, 3 to the non labeled vertices. If there are four non labeled vertices are exist then assign the labels 1, 2, 3, 2 to he non labeled vertices. If the number of non labeled vertices are three then assign the labels 1, 2, 3 to the non labeled vertices. If it is two then assign the labels 1, 2 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 1 only. Now we consider the vertices w_i. Assign the label to the vertices w_{6i+2}, w_{6i+5}, w_{6i+3} for all the values of $i = 0$, 1, 2,... Then assign the labels 3 to the vertices w_{6i+4} for $i = 0$, 1, 2, 3,... For all the values of $i = 1$, 2, 3... assign the label 3 to the vertices w_{6i} and w_{6i+1}. The edge condition is $e_f(0) = \frac{3n-4}{2}$ and $e_f(1) = \frac{3n-6}{2}$. The vertex condition is given in table table 14.
Case 3. The triangle starts from u_2 and end with u_n.
It is clear in this case $|V(DA(T_n))| = 2n - 1$ and $|E(DA(T_n))| = 3n - 3$. Assign the label 1 to the path vertices u_{12i+1} and u_{12i+2} for all the values of $i = 0$, 1, 2, 3,... Next we assign the label 2 to the path vertices $u_{12i+3}, u_{12i+4}, u_{12i+7}, u_{12i+8}$ and u_{12i+11} for $i = 0$, 1, 2, 3,... For

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 4, 10 \pmod{12}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 6 \pmod{12}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n-3}{3}$
$n \equiv 8 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-4}{3}$
$n \equiv 0 \pmod{12}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$	$\frac{2n-3}{3}$
$n \equiv 2 \pmod{12}$	$\frac{2n-1}{3}$	$\frac{2n-4}{3}$	$\frac{2n-1}{3}$

Table 14. Vertex label

the values of $i = 1, 2, 3,...$ assign the label 2 to the path vertices u_{12i}. Then we assign the label 3 to the path vertices $u_{12i+5}, u_{12i+6}, u_{12i+9}$ and u_{12i+10} for all the values of $i = 0, 1, 2, 3,...$ Now we consider the vertices v_i and w_i. Assign the labels 3, 2, 1, 1, 2, 3 to the first six vertices $v_1, v_2, v_3, v_4, v_5, v_6$ respectively. Then we assign the labels 3, 2, 1, 1, 2, 3 to the next six vertices $v_7, v_8, v_9, v_{10}, v_{11}, v_{12}$ respectively. Proceeding like this assign the label to the next six vertices and so on. The last six vertices $v_{n-5}, v_{n-4}, v_{n-3}, v_{n-2}, v_{n-1}, v_n$ are labeled by 3, 2, 1, 1, 2, 3. Consider the vertices w_i. Assign the label to the vertices w_i as in case 1. The vertex condition of this case is $e_f(0) = e_f(1) = \frac{3n-3}{2}$. The edge condition of this case is given by table 15.

Value of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 1 \pmod 6$	$\frac{2n+1}{3}$	$\frac{2n-2}{3}$	$\frac{2n-2}{3}$
$n \equiv 3 \pmod 6$	$\frac{2n}{3}$	$\frac{2n-3}{3}$	$\frac{2n}{3}$
$n \equiv 5 \pmod 6$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$	$\frac{2n-1}{3}$

Table 15. Vertex label

Case 4. The triangle starts from u_2 and end with u_{n-1}.
This case is similar to case 3. □

Finally we look into the graph double alternate quadrilateral snake. Double alternate quadrilateral snake DAQ_n consists of two alternate quadrilateral snake that have a common path. That is it is obtained from a path $u_1u_2...u_n$ joining u_i and u_{i+1} (alternatively) to new verticces v_i, x_i and w_i, y_i respectively and adding the edges v_iw_i and x_iy_i.

Theorem 2.9. *All double alternate quadrilateral snakes are 3-difference cordial graphs.*

Proof. **Case 1.** The squares starts from u_1 and end with u_n.
In this case $|V(DAQ_n)| = 3n$ and $|E(DAQ_n)| = 4n-1$. We consider the path vertices u_i. Assign the labels 1, 1, 3, 3 to the first four path vertices u_1, u_2, u_3, u_4. Then we sign the labels 1, 1, 3, 3 to the next four path vertices u_5, u_6, u_7, u_8. Continuing this way we assign the label to the next four vertices and so on. Clearly in this process the last vertex u_n received the label 3 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Now we move to the vertices v_i and w_i. Assign the labels 2, 1 to the first two vertices v_1 and v_2. Then we assign the labels 2, 1 to the next two vertices and so on. Proceeding like this, we assign the labels to the next two vertices and so on. Clearly the last verte v_{n-1} received the label 1 or 2 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Now we assign the labels 3, 2 to the two vertices w_1, w_2 respectively. Then we assign the labels 3, 2 to

the next two vertices w_3, w_4 respectively. Continuing this way we assign the label to the net two vertices and so on. Note that in this process the last vertex w_{n-1} receied the label 2 or 3 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices x_i and y_i. Assign the labels to the vertices x_i and y_i is same as assign the label to the vertices v_i and w_i. The vertex and edge condition of this case is $v_f(1) = v_f(2) = v_f(3) = n$ and $e_f(0) = 2n - 1$ and $e_f(1) = 2n$.

Case 2. The squares starts from u_1 and end with u_{n-1}.

In this case $|V(DA(Q_n))| = 3n - 4$ and $|E(DA(Q_n))| = 4n - 7$. Consider the path vertices u_i. Assign the label to the vertices u_i as in case 1. We move to the vertices v_i and w_i. Assign the labels 2, 2 to the vertices v_1 and w_2 respectively. Then assign the labels 2, 1 to the vertices v_2, v_3 respectively. Now we assign the labels 2, 1 to the next two vertices v_4, v_5 respectively. Continuing this process assign the label to the next two vertices and so on. Clearly the last vertex v_{n-1} received the label 1 or 2 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Assign the labels 3, 2 to the vertices w_2, w_3 respectively. Then assign the labels 3, 2 to the next two vertices w_4, w_5 respectively. Proceeding like this assign the label to the next two vertices and so on. Note that the last vertex w_n received the label 2 or 3 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices x_i and y_i. Assign the labels 2, 3 to the vertices x_1 and w_1 respectively. Now we assign the label 2, 1 to the vertices x_2, x_3. Then we assign the labels 2, 1 to the next two vertices x_4, x_5 respectively. Continuing like this we assign the label to the next two vertices and so on. Clearly the last vertex x_n labeled by the integers 1 or 2 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Assign the labels 3, 2 to the vertices y_2, y_3 respectively. Then we assign the labels 3, 2 to the next two vertices y_4, y_5 respectively. Continuing this process assign the label to the next two vertices and so on. Clearly the last vertex y_{n-1} received the label 2 or 3 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. The vertex and edge condition of this case is $v_f(1) = n - 2$ and $v_f(2) = v_f(3) = n - 1$ and $e_f(0) = 2n - 3$ and $e_f(1) = 2n - 4$.

Case 3. The squares starts from u_2 and end with u_n.

In this case $|V(DA(Q_n))| = 3n - 2$ and $|E(DA(Q_n))| = 4n - 4$. Consider the path vertices v_i. Assign the label 1 to the vertex u_1. Next we assign the labels 3, 3, 1, 1 to the next four vertices u_2, u_3, u_4, u_5 respectively. Then assign the labels 3, 3, 1, 1 to the net four vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this assign the label to the next four vertices and so on. Note that in this case the last verte u_n u_n received the label 3 or 1 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Next we move to the vertices v_i and w_i. Assign the labels 1, 2 to the vertices v_1, v_2 respectively. Then we assign labels 1, 2 to th next two vertices v_3, v_4 respectively. Proceeding this way we assign the next two verticces and so on. Clearly the last vertex v_n received the label 1 or 2 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Now we move to the vertices w_i. Assign the labels 2, 3 to the vertices w_1 and w_2 respectively. Then we assign the labels 2, 3 to the next two vertices w_3, w_4 respectively. Continuing this way assign the label to the next two vertices and so on. Note that the last vertex w_{n-1} received the label 2 or 3 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Consider the vertices x_i and y_i is same as assign the label to the vertices v_i and w_i. The vertex and edge condition of this case is $v_f(1) = n$ and $v_f(2) = v_f(3) = n - 1$ and $e_f(0) = e_f(1) = 2n - 2$. □

Acknowledgement

The authors wish to thank the anonymous referees for their careful reading and constructive comments on earlier version of this article, which resulted in better presentation of this article.

References

[1] I. Cahit, Cordial Graphs: A weaker version of Graceful and Harmonious graphs, *Ars Combin.,* **23** (1987) 201-207.

[2] J.A. Gallian, A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **19** (2016) #Ds6.

[3] F. Harary, Graph theory, *Addision Wesley*, New Delhi (1969).

[4] R. Hasni, S. Matarneh, A. Azaizeh, Some results on cordiality labeling of generalized Jahangir graph, *Indonesian Journal of Combinatorics*, **1**(2017),1-8.

[5] R. Ponraj, S. Sathish Narayanan and R. Kala, Difference cordial labeling of graphs, *Global Journal of Mathematical Sciences: Theory and Practical*, **5**(2013), 185-196.

[6] R. Ponraj and M. M. Adaickalam and R.Kala, k-difference cordial labeling of graphs, *International Journal of Mathematical Combinatorics*, **2**(2016), 121-131.

[7] R. Ponraj and M. M. Adaickalam, 3-difference cordial labeling of some union of graphs, *Palestine Journal of Mathematics*, **6(1)**(2017), 202-210.

[8] R. Ponraj and M. M. Adaickalam, 3-difference cordial labeling of cycle related graphs, *Journal of Algorithms and Computation*, **47**(2016), 1-10.

[9] R. Ponraj and M. M. Adaickalam, 3-difference cordiality of some graphs, *Palestine Journal of Mathematics*, **2**(2017), 141-148.

[10] R. Ponraj and M. M. Adaickalam, 3-difference cordial labeling of corona related graphs, *Submitted*.

[11] M. A. Seoud and S. M. Salman, On difference cordial graphs, *Mathematica Aeterna*, **5(1)** (2015), 105 - 124.