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Certain topological indices and polynomials for the semitotal-point graph and line graph of semi-total-point graph for Dutch windmill graph

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Abstract

Dutch windmill graph [8, 26], denoted by D_m^n is a well known family of graphs with cycles. Order and size of Dutch windmill graph are $(n-1)m+1$ and mn respectively. In this paper, we computed certain topological indices and polynomials $i.e.$ Zagreb polynomials, hyper Zagreb indices, Redefined Zagreb indices, modified first Zagreb index, Reduced second Zagreb index, Reduced Reciprocal Randić index, 1^{st} Gourava index, 2^{nd} Gourava index, 1^{st} hyper Gourava index, 2^{nd} hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in terms of M-polynomials $i.e.$ 1^{st} Zagreb index , 2^{nd} Zagreb index, Modified 2^{nd} Zagreb, Randić index, Reciprocal Randić index, Symmetric division index, Harmonic index, Inverse Sum indeg index, Augmented Zagreb index for the semitotal-point graph and line graph of semitotal-point graph for Dutch windmill graph.

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1. Definitions, Notations and Results

All the graphs in this paper are simple, finite and undirected. In a graph G, $V(G)$ and $E(G)$ are the sets of vertices and edges respectively. Let $d_G(u)$ denotes the degree of a vertex u. Topological indices have been found to be useful in establishing relation between the structure and the properties of molecules. Topological indices mainly used in Quantitative Structure Property Relationship (QSPR) and Quantitative Structure Activity Relationships (QSAR)[5]. Some topological indices are degree based and some are distance based.

The *Zagreb indices* were introduced more than thirty years ago by Gutman and Trinajstić [14]. After ten years, Balaban *et.al* named them *Zagreb group index*, presented by M_1 and M_2. Later it was abbreviated to *Zagreb index* [17], where M_1 and M_2 represents *first Zagreb index* and *second Zagreb index* respectively. If d_u and d_v are the degrees of vertices u, v for simple graph G, then *first Zagreb index* [17, 4] is defined as

$$\begin{aligned} M_1(G) &= \sum_{p \in V(G)} (d_G(p))^2 \\ &= \sum_{pq \in E(G)} d_G(p) + d_G(q) \end{aligned}$$

Second Zagreb index is defined as

$$M_2(G) = \sum_{pq \in E(G)} d_G(p) d_G(q)$$

Third Zagreb index introduced by Fath-Tabar [23] in 2011. Which is denoted by $ZG_3(G)$ for a simple graph G and defined as

$$ZG_3(G) = \sum_{pq \in E(G)} | d_p - d_q |$$

Authors proposed the concept of *First, Second and Third Zagreb polynomials* [23], for a simple graph G defined as

$$ZG_1(G, x) = \sum_{pq \in E(G)} x^{d_p + d_q} \tag{1}$$

$$ZG_2(G, x) = \sum_{pq \in E(G)} x^{d_p d_q} \tag{2}$$

$$ZG_3(G, x) = \sum_{pq \in E(G)} x^{|d_p - d_q|} \tag{3}$$

The *Modified Zagreb index* [15] is also an important degree based graph invariant. *First modified Zagreb index* for a graph H is denoted by ${}^m M_1(G)$, defined as

$$\begin{aligned} {}^m M_1(G) &= \sum_{q \in V(G)} \frac{1}{(d_q)^2} \\ &= \sum_{pq \in E(G)} \frac{1}{d_p + d_q}. \end{aligned} \tag{4}$$

Similarly *second modified Zagreb indices* is denoted by ${}^{m}M_2(G)$, defined as

$${}^{m}M_2(G) = \sum_{pq \in E(G)} \frac{1}{d_p d_q}.$$

Shirdel *et.al*[27] brought into notice a new degree based graph invariant named as *hyper-Zagreb index*, which is defined as

$$HM(G) = \sum_{pq \in E(G)} (d_p + d_q)^2 \quad (5)$$

Forgotten topological index is also a degree based topological index, denoted by $F(G)$ for simple graph G. It was presented in [10], defined as

$$\begin{aligned} F(G) &= \sum_{q \in V(G)} (d_q)^3 \\ &= \sum_{pq \in E(G)} [(d_p)^2 + (d_q)^2] \end{aligned} \quad (6)$$

Forgotten polynomial for a graph G is defined as

$$F(G, x) = \sum_{pq \in E(G)} x^{[(d_p)^2 + (d_q)^2]} \quad (7)$$

Randić connectivity index was put forward by Randić in 1975, defined as

$$\begin{aligned} R_{-1/2}(G) &= \sum_{pq \in E(G)} (d_p d_q)^{-1/2} \\ &= \sum_{pq \in E(G)} \frac{1}{\sqrt{d_p d_q}} \end{aligned} \quad (8)$$

and extended to *general Randić connectivity index* [16]. If α is a real number, then it is defined as

$$R_\alpha(G) = \sum_{pq \in E(G)} (d_p d_q)^\alpha$$

In 2014, I. Gutman $et.al.$ [13] proposed *reciprocal Randić, reduced second Zagreb* and *reduced reciprocal Randić index*, these are degree based graph invariants, defined as

$$RR(G) = \sum_{pq \in E(G)} \sqrt{d_p d_q}$$

$$RM_2(G) = \sum_{pq \in E(G)} (d_p - 1)(d_q - 1) \quad (9)$$

and

$$RRR(G) = \sum_{pq \in E(G)} \sqrt{(d_p - 1)(d_q - 1)} \quad (10)$$

Harmonic index for a graph G defined as

$$H(G) = \sum_{pq \in E(G)} \frac{2}{d_p + d_q}$$

Many researchers are working on these graph invariants as only one index does not fully describe chemical properties of a chemical structure. Recently in 2017, V. R. Kulli proposed many new graph invariants [18, 19, 20, 21], known as family of Gourava indices, defined as

$$\textit{First Gourava Index} = G_1O(G) = \sum_{pq \in E(G)} [d_p + d_q + d_p d_q] \tag{11}$$

and

$$\textit{Second Gourava Index} = G_2O(G) = \sum_{pq \in E(G)} [(d_p + d_q)(d_p d_q)] \tag{12}$$

respectively.
We can rewrite second Gourava index as

$$= \sum_{pq \in E(G)} [d_p^2 d_q + d_q^2 d_p] \tag{13}$$

Product Connectivity Gourava index is denoted by PGO and defined as

$$PGO(G) = \sum_{pq \in E(G)} \frac{1}{\sqrt{(d_p + d_q)(d_p d_q)}}. \tag{14}$$

Sum connectivity Gourava index is given by

$$SGO(G) = \sum_{pq \in E(G)} \frac{1}{\sqrt{(d_p + d_q) + (d_p d_q)}}. \tag{15}$$

Hyper-Gourava indices $i.e.$ *first and second hyper Gourava index* are defined as

$$HGO_1(G) = \sum_{pq \in E(G)} [(d_p + d_q) + (d_p d_q)]^2 \tag{16}$$

and

$$HGO_2(G) = \sum_{pq \in E(G)} [(d_p + d_q)(d_p d_q)]^2 \tag{17}$$

respectively. There are some new degree based graph invariants, which plays an important role in chemical graph theory. These topological indices are quite useful for determining total surface area and heat formation of some chemical compounds. These graph invariants are as follows *Symmetric division index*

$$SDD(G) = \sum_{pq \in E(G)} \left\{ \frac{min(d_p, d_q)}{max(d_p, d_q)} + \frac{max(d_p, d_q)}{min(d_p, d_q)} \right\}$$

inverse sum index

$$I(G) = \sum_{pq \in E(G)} \frac{d_p d_q}{d_p + d_q}$$

and *augmented Zagreb index* for a graph H

$$A(G) = \sum_{pq \in E(G)} \left\{ \frac{d_p d_q}{d_p + d_q - 2} \right\}^3$$

In 2009, Zhou and Trinajstić proposed *sum-connectivity index* defined as

$$\chi_{-1/2}(G) = \sum_{pq \in E(G)} [(d_p + d_q)]^{-1/2} \tag{18}$$

In 2013, Ranjini [25] introduced *redefined Zagreb indices* $i.e.$ *redefined first, second* and *third Zagreb indices* of a graph G. For a graph G, these indices were computed by following formulas $i.e.$ *redefined first Zagreb index*,

$$ReZG_1(G) = \sum_{pq \in E(G)} \frac{d_p + d_q}{d_p d_q} \tag{19}$$

redefined second Zagreb index

$$ReZG_2(G) = \sum_{pq \in E(G)} \frac{d_p d_q}{d_p + d_q} \tag{20}$$

and *redefined third Zagreb index*

$$ReZG_3(G) = \sum_{pq \in E(G)} (d_p d_q)(d_p + d_q) \tag{21}$$

As *algebraic polynomials* play quite significant role in determining the bioactivity of chemical compounds, so some algebraic polynomials are also considered in this work. *M-polynomial* [1] is also one of these useful algebraic polynomials, defined as

$$M(G, x, y) = \sum_{\delta \leq i \leq j \leq \Delta} m_{ij}(H) x^i y^j \tag{22}$$

where $\delta = min\{d_G(p)\}$, $\Delta = max\{d_G(p)\}$ and $m_{ij} = |E(G)|$ for $p, q \in V(G)$ of graph G. Some of the topological indices are directly determined by $M - polynomial$ for $x = y = 1$. These topological indices are stated as (refer to [1])

First Zagreb index

$$M_1(G) = (D_x + D_y)(M(G; x, y))_{x=y=1} \tag{23}$$

Second Zagreb index

$$M_2(G) = (D_x D_y)(M(G; x, y))_{x=y=1} \tag{24}$$

Modified second Zagreb index

$$^{m}M_2(G) = (S_x S_y)(M(G; x, y))_{x=y=1} \tag{25}$$

Randić index

$$R_\alpha(G) = (S_x^\alpha S_y^\alpha)(M(G; x, y))_{x=y=1} \tag{26}$$

Inverse Randić index

$$RR(G) = (D_x^\alpha D_y^\alpha)(M(G; x, y))_{x=y=1} \tag{27}$$

Symmetric division index

$$SDD(G) = (D_x S_y + D_y S_x)(M(G; x, y))_{x=y=1} \tag{28}$$

Harmonic index

$$H(G) = 2S_x J(M(G; x, y))_{x=y=1} \tag{29}$$

Inverse sum index

$$I(G) = S_x J D_x D_y(M(G; x, y))_{x=y=1} \tag{30}$$

and *Augmented Zagreb index*

$$A(G) = S_x^3 Q_{-2} D_x^3 D_y^3(M(G; x, y))_{x=y=1} \tag{31}$$

Where

$$D_x M(G(x, y)) = x\frac{\partial(M(G(x, y)))}{\partial x}$$

$$D_y M(G(x, y)) = y\frac{\partial(M(G(x, y)))}{\partial y}$$

$$S_x M(G(x, y)) = \int_0^x \frac{M(G(t, y))}{t} dt$$

$$S_y M(G(x, y)) = \int_0^x \frac{M(G(x, t))}{t} dt$$

$$JM(G(x, y)) = M(G(x, x))$$

and

$$Q_\alpha M(G(x, y)) = x^\alpha M(G(x, y)).$$

Since last thirty years, many scholars and researchers have been working on *composite graphs*. There are various graph operations which are applied directly on simple graphs to study their properties under these operations. Many authors computed several topological indices of some graph operations and line graph of these graph operations for certain families [24, 11, 7, 6, 22, 2, 28], *e.g.* composition, disjunction, Cartesian product, corona product, indu-bala product and wreath product of two graphs.

Subdivision S(G) [7, 6, 22, 2, 28] of a graph is acquired by embedding a vertex referred as the *white vertex* into each edge of *G*. Two black vertices are *related* in $S(G)$ if they are adjacent in

G. So *semitotal-point graph* $R(G)$ is obtained from $S(G)$ by joining each pair of *related* black vertices. *Line graph* $L(G)$ is another graph operation, that was thoroughly considered in the precise starting work of the structural chemistry. In 1981, Bertz [3] introduced first topological index based on line graph. Many authors computed the line graph of certain families of graphs in [12, 9, 24, 11, 2]. Many authors computed several topological indices for these four graph operations. M. Faisal $et.al.$ [24] computed ABC_4 and GA_5 indices of the line graph of tadpol, wheel and ladder graphs using the notion of subdivision. In [11] Y.Gao $et.al.$ provided the formulas for some topological indices of line graphs of the subdivision graphs of Nanotube and Nanotorus of TUC_4C_8. M. Reza $et.al.$ [7] considered Wiener index and Hosoya polynomial of the line graph of the wheel graphs using the concept of subdivision and explored new results. He [6] also executed the exact values of Schultz and modified Schultz polynomial of the subdivision graph and line graph subdivision graph for wheel. M. Ajmal $et.al.$ [2] worked on forgotten polynomial and forgotten index of line graphs of Banana tree graphs, Firecracker graph and subdivision graphs.

2. Certain Topological Indices and polynomials of $R(D_n^m)$

In this section, we compute Zagreb polynomials, hyper Zagreb index, Redefined Zagreb indices, modified first Zagreb index, Reduced second Zagreb index, Reduced Reciprocal Randić index, 1^{st} Gourava index, 2^{nd} Gourava index, 1^{st} hyper Gourava index, 2^{nd} hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in terms of M-polynomials $i.e.$ 1^{st} Zagreb index, 2^{nd} Zagreb index, Modified 2^{nd} Zagreb index, Randić index, Reciprocal Randić index, Symmetric division index, Harmonic index, Inverse Sum indeg index, Augmented Zagreb index for the semitotal-point graph of Dutch windmill graph $i.e.$ $R(D_n^m)$. As the semitotal-point graph of Dutch windmill graph is shown in figure 2.

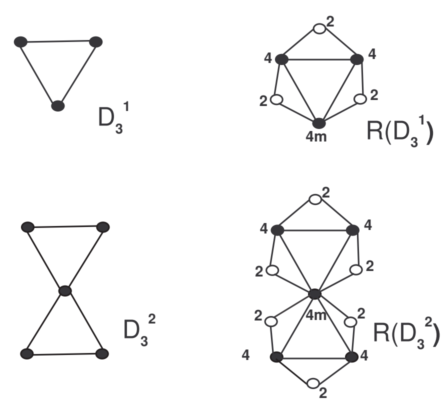

Figure 1. Semitotal-point graph of Dutch windmill graph, $i.e.$ $R(D_n^m)$

Theorem 2.1. *Let G be the Semitotal-point graph of Dutch windmill graph, $i.e.$ $R(D_n^m)$. Then the forgotten polynomial and Zagreb polynomials are given by*

$$F(G,x) = 2m(x^{16m^2+4} + x^{16m^2+16}) + (mn-2m)x^2 + (2mn-2m)x^{20}$$
$$ZG_1(G,x) = 2mx^{4m}x^2(x^2+1) + mnx^6(x^2+2) - 2mx^6(x^2+1)$$
$$ZG_2(G,x) = 2mx^{8m}(1+x^{8m}) + mn(2+x^8) - 2mnx^8(1+x^8)$$
$$ZG_3(G,x) = 2m(x^{4m-2} + x^{4m-4}) + mn(1+2x^2) - 2m(1+x^2)$$

Proof. Consider the Semitotal-point graph of Dutch windmill graph, denoted by $R(D_n^m)$. There are $2mn-(m-1)$ total number of vertices and $3mn$ total number of edges in $R(D_n^m)$. $R(D_n^m)$ contains the vertices with degrees $4m$, 4 and 2. Degree based edge partition of $R(D_n^m)$ is denoted by $E_{(d_p,d_q)}$, where $pq \in E(R(D_n^m))$. The edge partition $E_{(4m,2)}$ contains $2m$ edges for $d_p = 4m$ and $d_q = 2$, the edge partition $E_{(4m,4)}$ contains $2m$ edges for $d_p = 4m$ and $d_q = 4$, edge partition $E_{(4,4)}$ contains $(n-2)m$ edges for $d_p = 4$ and $d_q = 4$ and the edge partition $E_{(2,4)}$ contains $(2n-2)m$ edges for $d_p = 2$ and $d_q = 4$, shown in Table 1

Table 1. Edge partition of $R(D_n^m)$

Edges of type	Number of edges
$E_{(4m,2)}$	$2m$
$E_{(4m,4)}$	$2m$
$E_{(4,4)}$	$(n-2)m$
$E_{(2,4)}$	$(2n-2)m$

Forgotten polynomial :

$$\mathbf{F(G,x)} = \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{(d_p)^2+(d_q)^2}}$$
$$= 2m(x^{16m^2+4} + x^{16m^2+16}) + (mn-2m)x^2 + (2mn-2m)x^{20}$$

1^{st} **Zagreb polynomial** :

$$\mathbf{ZG_1(G,x)} = \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{d_p+d_q}}$$
$$= \sum x^{4m+2} + \sum x^{4m+4} + \sum x^{4+4} + \sum x^{2+4}$$
$$= 2m(x^{4m+2}) + 2mx^{4m+4} + (nm-2m)x^{4+4} + (2nm-2m)x^{2+4}$$
$$= 2mx^{4m}x^2 + 2mx^{4m}x^4 + mnx^8 - 2mx^8 + 2mnx^6 - 2mx^6$$
$$= 2mx^{4m}x^2(x^2+1) + mnx^6(x^2+2) - 2mx^6(x^2+1)$$

2^{nd} **Zagreb polynomial**:

$$\begin{aligned}
\mathbf{ZG_2(G,x)} &= \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{d_p d_q}} \\
&= \sum x^{4m\times 2} + \sum x^{4m\times 4} + \sum x^{4\times 4} + \sum x^{2\times 4} \\
&= 2mx^{8m} + 2mx^{16m} + mnx^{16} - 2mx^{16} + 2mnx^{8} - 2mx^{8} \\
&= 2mx^{8m}(1 + x^{8m}) + mnx^{8}(2 + x^{8}) - 2mx^{8}(1 + x^{8}) \\
&= 2mx^{8}(1 + x^{8}) + (1 + x^{8})x^{8}m[n - 2] \\
&= 2mx^{4}(x^{m} + n - 1)
\end{aligned}$$

3^{rd} **Zagreb polynomial**:

$$\mathbf{ZG_3(G,x)} = \sum_{\mathbf{pq\in E(G)}} \mathbf{x^{|d_p - d_q|}}$$

$$\begin{aligned}
&= \sum x^{4m-2} + \sum x^{4m-4} + \sum x^{4-4} + \sum x^{2-4} \\
&= 2mx^{4m-2} + 2mx^{4m-4} + mnx^{0} - 2mx^{0} + 2mnx^{2} - 2mx^{2} \\
&= 2m(x^{4m-2} + x^{4m-4}) + mn - 2m + 2mnx^{2} - 2mx^{2} \\
&= 2m(x^{4m-2} + x^{4m-4}) + mn(1 + 2x^{2}) - 2m(1 + x^{2})
\end{aligned}$$

□

Theorem 2.2. *Let G be the semitotal-point graph of a Dutch Windmill graph. Then*

$$\begin{aligned}
HM(G) &= 64m^3 + 96m^2 - 160m + 136mn \\
ReZG_1(G) &= 1 - m + 2mn \\
ReZG_2(G) &= \frac{16m^2}{4m+2} + \frac{32m}{4m+4} + \frac{14}{3}mn - \frac{20}{3}m \\
ReZG_3(G) &= 192m^3 + 160m^2 - 352m + 224mn \\
{}^{m}M_2(G) &= m[\frac{1}{2m+1} + \frac{1}{2m+2} - \frac{7}{12}] + mn\frac{11}{24} \\
RM_2(G) &= 32m^2 - 32m + 15mn \\
RRR(G) &= 2m\sqrt{4m-1} + 2m\sqrt{12m-3} + 3mn - 6m + 2mn\sqrt{(3)} - 2m\sqrt{(3)} \\
GO_1(G) &= 64m^2 - 64m + 52mn \\
GO_2(G) &= 192m^3 + 160m^2 - 352m + 224mn \\
HGO_1 &= 1088m^3 + 416m^2 - 1504m + 968mn \\
HGO_2 &= 10240m^5 + 18432m^4 + 8704m^3 - 37376m + 20992mn \\
PGO(G) &= m\frac{1}{\sqrt{8m^2+4m}} + m\frac{1}{4\sqrt{m^2+m}} + mn\frac{1}{8\sqrt{2}} - m\frac{1}{4\sqrt{2}} + mn\frac{1}{2\sqrt{3}} - m\frac{1}{2\sqrt{3}}
\end{aligned}$$

Proof. Apply Formulas (5), (19), (20), (21), (25), (9), (10), (11),(13), (16), (17), (1), (15) and (6) to the edge partitions shown in Table 1 to get the required results. □

Theorem 2.3. *Let G be the semitotal-point graph of a Dutch Windmill graph. Then M-polynomial and certain topological indices in term of M-polynomial are*

$$\begin{aligned}
M(G;x,y) &= 2mx^{4m}y^2(1+y^2) + mx^4y^4(n-2) + 2mx^2y^4(n-1)\\
M_1(G) &= 16m^2 + 20mn - 16m\\
M_2(G) &= 48m^2 + 32mn - 48m\\
{}^mM_2(G) &= \frac{3}{8} + \frac{5mn}{16} - \frac{3m}{8}\\
R_\alpha(G) &= \frac{2m}{(8m)^\alpha} + \frac{2m}{(16m)^\alpha} + \frac{m(n-2)}{(16^\alpha)} + \frac{2m(n-1)}{(8^\alpha)}\\
RR_\alpha(G) &= 2m(8m)^\alpha + 2m(16m)^\alpha + (16^\alpha)(n-2) + (2n-2)m(8^\alpha)\\
SDD(G) &= 6m^2 + 7mn - 9m + 3\\
H(G) &= \frac{4m}{4m+2} + \frac{4m}{4m+4} - \frac{7m}{12} + \frac{11m}{24}\\
I(G) &= \frac{16m^2}{4m+2} + \frac{32m^2}{4m+4} + \frac{16mn}{8} - \frac{32m}{8} + \frac{16mn}{6} - \frac{16m}{6}\\
A(G) &= 16m + \frac{8192m^4}{(4m+2)^3} + \frac{4096(n-2)}{216} + \frac{1024m(n-1)}{64}
\end{aligned}$$

Proof. Apply Formulas (22), (23), (24), (25), (26), (27), (28), (29), (30) and (31) to the edge partitions shown in the Table 1 to get the required results. □

3. Certain Topological Indices and polynomials of $L(R(D_n^m))$

In this section, we compute Zagreb polynomials, hyper Zagreb, Redefined Zagreb indices, modified first Zagreb, Reduced second Zagreb, Reduced Reciprocal Randić, 1st Gourava index, 2nd Gourava index, 1st hyper Gourava index, 2nd hyper Gourava index, Product connectivity Gourava index, Sum connectivity Gourava index, Forgotten index, Forgotten polynomials, M-polynomials and some topological indices in term of the M-polynomials i.e 1st zagreb index, 2nd Zagreb index, Modified 2nd Zagreb, Randić index, Reciprocal Randić index, Symmetric division, Harmonic index, Inverse Sum index, Augmented Zagreb index for the line graph of semitotal point graph of Dutch windmill graph. As the line graph of semitotal point graph of Dutch windmill graph is shown in figure 3.

Theorem 3.1. *Let G be the Line graph of semitotal point graph of a Dutch windmill graph i.e $L(R(D_n^m))$. Then forgotten polynomial and Zagreb polynomials are given by*

$$\begin{aligned}
F(G,x) &= m^2[2x^{32m^2+32m+8} + 4x^{32m^2+5} + 2x^{32m^2}] + m[x^{16m^2+16m+20} - x^{32m^2+32m+8}\\
&\quad + 2x^{16m^2+16m+40} - x^{32m^2} + 2x^{16m^2+16} - 3x^{72} - 3x^{32} - 3x^{52}] + mn[x^{72} + 2x^{32} + 4x^{52}]\\
ZG_1(G,x) &= m^2[2x^{8m+4} + 4x^{8m+4} + 2x^{8m}] + m[4x^{4m+6} - x^{8m+4} + 2x^{4m+8} - x^{8m} + 2x^{4m+4}\\
&\quad - 3x^{12} - 3x^8 - 8x^{10}] + mn[2x^8 + 4x^{10} + x^{12}]\\
ZG_2(G,x) &= m^2[2x^{16m^2+16m+4} + 4x^{16m^2+16m+4} + 3x^{16m^2}] + m[4x^{64m+32} - x^{16m^2+16m+4}\\
&\quad + 2x^{24m+12} - x^{16m^2} + 2x^{16m} - 3x^{36} - 3x^{16}] + mn[x^{16} + x^{36} + 4x^{24}]\\
ZG_3(G,x) &= 4m^2(1+x^2) + m[4x^{4m-4} + 2x^{4m-2} + 2x^{4m-4} - 8x^2 - 8] + mn[3 + 4x^2]
\end{aligned}$$

Proof. Consider the Line graph of semitotal-point graph of Dutch windmill graph, denoted by $L(R(D_n^m))$. The total no of vertices and edges in $L(R(D_n^m))$ are $3mn$ and $8m^2 + 7mn - 8m$. $R(D_n^m)$ contains the vertices with degrees $4m + 2$, $4m$,4, and 6. We partition the edges of $L(R(D_n^m))$ based on the edges of type $E_{(d_u,d_v)}$, where $uv \in E(R(D_n^m)))$. The edge partition $E_{(4m+2,4m+2)}$ contains $m(2m-1)$ edges for $d_u = 4m+2$ and $d_v = 4m+2$,the edge partition $E_{(4m+2,4m)}$ contains $4m^2$ edges for $d_u = 4m+2$ and $d_v = 4m$, the edge partition $E_{(4m+2,4)}$ contains $4m$ edges for $d_u = 4m+2$ and $d_v = 4$, the edge partition $E_{(4m+2,6)}$ contains $2m$ edges for $d_u = 4m+2$ and $d_v = 6$, the edge partition $E_{(4m,4m)}$ contains $m(2m-1)$ edges for $d_u = 4m$ and $d_v = 4m$, the edge partition $E_{(4m,4)}$ contains $2m$ edges for $d_u = 4m$ and $d_v = 4$, the edge partition $E_{(6,6)}$ contains $(n-3)m$ edges for $d_u = 6$ and $d_v = 6$, the edge partition $E_{(4,4)}$ contains $(2n-3)m$ edges for $d_u = 4$ and $d_v = 4$, the edge partition $E_{(6,4)}$ contains $(4n-8)m$ edges for $d_u = 6$ and $d_v = 4$ shown in Table 2

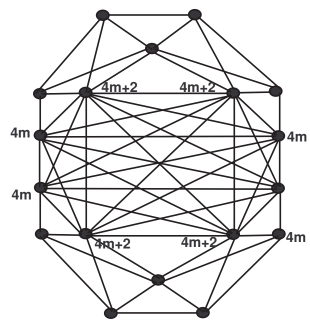

Figure 2. Line graph of Semitotal-point of Dutch windmill graph $i.e.$ $L(R(D_3^2))$

Table 2. Edge partition of $L(R(D_n^m))$

Edges of type	Number of edges
$E_{(4m+2,4m+2)}$	$m(2m-1)$
$E_{(4m+2,4m)}$	$4m^2$
$E_{(4m+2,4)}$	$4m$
$E_{(4m+2,6)}$	$2m$
$E_{(4m,4m)}$	$m(2m-1)$
$E_{(4m,4)}$	$2m$
$E_{(6,6)}$	$(n-3)m$
$E_{(4,4)}$	$(2n-3)m$
$E_{(6,4)}$	$(4n-8)m$

$$\begin{aligned}
\mathbf{F(G,x)} &= \sum_{\mathbf{uv\in E(G)}} \mathbf{x^{(d_u)^2+(d_v)^2}} \\
&= 2m^2x^{(4m+2)^2+(4m+2)^2} - mx^{(4m+2)^2+(4m+2)^2} + 4m^2x^{(4m+2)^2+(4m)^2} + 4mx^{(4m+2)^2+(4)^2} \\
&+ 2mx^{(4m+2)^2+(6)^2} + 2m^2x^{(4m)^2+(4m)^2} - mx^{(4m)^2+(4m)^2} + 2mx^{(4m)^2+(4)^2} + mnx^{(6)^2+(6)^2} \\
&- 3mx^{(6)^2+(6)^2} + 2mnx^{(4)^2+(4)^2} - 3mx^{(4)^2+(4)^2} + 4mnx^{(6)^2+(4)^2} - 3mx^{(6)^2+(4)^2} \\
&= 2m^2x^{32m^2+32m+8} - mx^{32m^2+32m+8} + 4m^2x^{32m^2+16m+4} + 2mx^{16m^2+16m+40} + 2m^2x^{32m^2} \\
&- mx^{32m^2} + 2mx^{16m^2+16} + mnx^{72} - 3mx^{72} + 2mnx^{32} - 3mx^{32} + 4mnx^{52} - 3mx^{52} \\
&+ 2mx^{16m^2+16m+20} \\
&= m^2[2x^{32m^2+32m+8} + 4x^{32m^2+4+16m} + 2x^{32m^2}] + m[x^{16m^2+16m+20} - x^{32m^2+32m+8} \\
&+ 2x^{16m^2+16m+40} - x^{32m^2} + 2x^{16m^2+16} - 3x^{72} - 3x^{32} - 3x^{52}] + mn[x^{72} + 2x^{32} + 4x^{52}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{ZG_1(G,x)} &= \sum_{\mathbf{uv\in E(G)}} \mathbf{x^{d_u+d_v}} \\
&= \sum x^{4m+2+4m+2} - \sum x^{4m+2+4m+2} + \sum x^{4m+2+4m} + \sum x^{4m+2+4} + \sum x^{4m+2+6} \\
&+ \sum x^{4m+4m} + \sum x^{4m+4} + \sum x^{6+6} + \sum x^{4+4} \\
&+ \sum x^{6+4} \\
&= m^2[2x^{8m+4} + 4x^{8m+4} + 2x^{8m}] + m[4x^{4m+6} - x^{8m+4} + 2x^{4m+8} - x^{8m} \\
&+ 2x^{4m+4} - 3x^{12} - 3x^{8} - 8x^{10}] + mn[2x^{8} + 4x^{10} + x^{12}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{ZG_2(G,x)} &= \sum_{\mathbf{uv\in E(G)}} \mathbf{x^{d_ud_v}} \\
&= 2m^2x^{(4m+2)(4m+2)} - mx^{(4m+2)(4m+2)} + m^2x^{(4m+2)(4m)} + 4mx^{(4m+2)(4)} + 2mx^{(4m+2)(6)} \\
&+ 2m^2x^{(4m)(4m)} - mx^{(4m)(4m)} + 2mx^{(4m)(4)} + mnx^{(6)(6)} - 3mx^{(6)(6)} + 2mnx^{(4)(4)} \\
&- 3mx^{(4)(4)} + 4mnx^{(6)(4)} + 8mx^{(6)(4)} \\
&= m^2[2x^{16m^2+16m+4} + 4x^{16m^2+16m+4} + 3x^{16m^2}] + m[4x^{64m+32} - x^{16m^2+16m+4} \\
&+ 2x^{24m+12} - x^{16m^2} + 2x^{16m} - 3x^{36} - 3x^{16}] + mn[x^{16} + x^{36} + 4x^{24}]
\end{aligned}$$

$$\begin{aligned}
\mathbf{ZG_3(G,x)} &= \sum_{\mathbf{uv\in E(G)}} \mathbf{x^{|d_u-d_v|}} \\
&= 2m^2x^{4m+2-4m-2} - mx^{4m+2-4m-2} + 4m^2x^{4m+2-4m} + 4mx^{4m+2-4} + 2mx^{4m+2-6} \\
&+ 2m^2x^{4m-4m} - mx^{4m-4m} + 2mx^{4m-4} + mnx^{6-6} - 3mx^{6-6} + 2mnx^{4-4} \\
&- 3mx^{4-4} + 4mnx^{6-4} - 8mx^{6-4} \\
&= 4m^2 + 4m^2x^2 + m[4x^{4m-2} + 2x^{4m-4} + 2x^{4m-4} - 8x^2 - 8] + mn[3 + 4x^2] \\
&= 4m^2(1 + x^2) + m[4x^{4m-2} + 2x^{4m-4} + 2x^{4m-4} - 8x^2 - 8] + mn[3 + 4x^2]
\end{aligned}$$

□

Theorem 3.2. *Let G be the Line graph of semitotal point graph of a Dutch Windmill graph. Then*

$$
\begin{aligned}
HM(G) &= 384m^4 + 32m^3 + 48m^2 - 1280m + 762mn \\
ReZG_1(G) &= \frac{(2m^2-m)(2m+1)}{4m^2+4m+1} + \frac{(4m^3+m^2)}{2m^2+m} + \frac{(2m^2+3m)}{(2m+1)} + \frac{(2m^2+4m)}{(6m+3)} \\
&+ \frac{(2m^2-m)}{2m} + \frac{(m+1)}{2} + \frac{(n-3)m}{3} + \frac{(2n-3)m}{2} + \frac{(n-2)5m}{3} \\
ReZG_2(G) &= 8m^4 + m^3[\frac{8}{2m+1} - \frac{4}{m+2} + \frac{32}{4m+1} + 4] + m^2[\frac{4}{4m+2} - \frac{4}{2m+1} + \frac{8}{m+1} \\
&+ \frac{16}{4m+1} + \frac{16}{2m+3} + \frac{12}{m+2} - 2] + m[-15 - \frac{1}{m+1} + \frac{16}{2m+3} + \frac{6m}{m+2}] + \frac{83mn}{5} \\
ReZG_3(G) &= 768m^5 + 512m^4 + 512m^3 + 928m^2 - 3424m + 1648mn \\
{}^{m}M_2(G) &= m^2[\frac{1}{2m+1} + \frac{2}{4m+1}] + m[\frac{2}{2m+3} - \frac{1}{8m+4} + \frac{1}{2m+4} + \frac{1}{2m+2} - \frac{47}{40}] \\
&+ \frac{13mn}{20} - \frac{1}{8} \\
RM_2(G) &= 128m^4 - 32m^3 + 116m^2 - 208m + 103mn \\
RRR(G) &= m^2[2\sqrt{16m^2+8m+1} + 4\sqrt{16m^2-1} + 2\sqrt{16m^2-8m+1} - \sqrt{16m^2+8+1}] \\
&+ m[4\sqrt{12m+3} + 2\sqrt{20m+5} - \sqrt{16m^2-8m+1} + 2\sqrt{12m-3} - 8\sqrt{5} - 24] \\
&+ mn[4\sqrt{15} + 11] \\
GO_1(G) &= 128m^4 + 96m^3 + 168m^2 - 408m + 232mn \\
GO_2(G) &= 768m^5 + 512m^4 + 512m^3 + 928m^2 - 3424m + 1648mn \\
HGO_1 &= 1536m^6 + 3584m^5 + 2048m^4 + 4096m^3 + 4560m^2 - 16338m + 8080mn \\
HGO_2 &= 449792mn - 1069624m + 32768m^8 - 16384m^6 + 128m^3 + 128m^2 \\
&+ (256m^4 + 192m^3 + 32m^2)^2 + m(128m^2 + 256m + 96)^2 + 2m(96m^2 + 240m + 96)^2 \\
&+ (2m^2 - m)(128m^3 + 192m^2 + 96m + 16)^2 \\
PGO(G) &= \frac{m^2}{64m^3+96m^2+48m+8} + \frac{m}{16m^2+32m+12} + \frac{m}{32m^2+24m+4} \\
&+ \frac{m}{16m^2+32m+12} + \frac{m}{48m^2+122m+48} + \frac{1}{64m} - \frac{1}{128m^2} + \frac{1}{32m+32} \\
&+ \frac{4mn}{432} - \frac{m}{144} + \frac{mn}{60} - \frac{m}{30} + \frac{12mn}{3072} - \frac{8m}{3072} \\
SGO(G) &= \frac{m^2}{8m^2+12m+4} - \frac{m}{16m^2+24m+8} + \frac{2m^2}{8m^2+8m+1} + \frac{2m}{10m+7} + \frac{m}{14m+10} \\
&+ \frac{m}{4+8m} - \frac{1}{8+16m} + \frac{m}{10m+2} + \frac{mn}{48} - \frac{m}{16} + \frac{mn}{12} - \frac{m}{8} + \frac{mn}{6} - \frac{m}{3072} \\
F(G) &= 176m^4 + 208m^3 + 112m^2 - 672m + 344mn
\end{aligned}
$$

Proof. Apply Formulas (5), (19), (20), (21), (25), (9), (10), (11),(13), (16), (17), (1), (15) and (6) to the edge partitions shown in Table 2 to get the required results. □

Theorem 3.3. *Let G be the Line graph of Semitotal-point graph of a Dutch Windmill graph i.e*

$L(R(D_n^m))$. *Then M-polynomial and certain topological indices in terms of M-polynomial are*

$$\begin{aligned}
M(G;x,y) &= 2m^2x^{4m+2}y^{4m+2} - mx^{4m+2}y^{4m+2} + 4m^2x^{4m+2}y^{4m} + 4mx^{4m+2}y^4 + 2mx^{4m+2}y^6 \\
&+ 2m^2x^{4m}y^{4m} - mx^{4m}y^{4m} + 2mx^{4m}y^4 + mnx^6y^6 - 3mx^6y^6 + 2mnx^4y^4 \\
&- 3mx^4y^4 + 4mnx^6y^4 - 8mx^6y^4 \\
M_1(G) &= 64m^3 + 32m^2 - 96m + 68mn \\
M_2(G) &= 128m^4 + 32m^3 + 136m^2 - 296m + 164mn \\
{}^mM_2(G) &= \frac{m^2}{(2m+1)(4m+2)} - \frac{m}{(4m+2)^2} + \frac{2m}{4m+2} + \frac{m}{(12m+6)} + \frac{m}{(8m)} - \frac{1}{(16m)} \\
&+ \frac{m}{(8m)} + \frac{mn}{(36)} + \frac{m}{(12)} + \frac{mn}{(8)} - \frac{3m}{(16)} + \frac{mn}{6} - \frac{m}{3} \\
R_\alpha(G) &= \frac{2m^2}{(4m+2)^\alpha(4m+2)^\alpha} - \frac{m}{(4m+2)^\alpha(4m+2)^\alpha} + \frac{4m^2}{(4m)^\alpha(4m+2)^\alpha} + \frac{4m}{(4)^\alpha(4m+2)^\alpha} \\
&+ \frac{2m}{(6)^\alpha(4m+2)^\alpha} + \frac{2m^2}{(4m)^\alpha(4m)^\alpha} - \frac{m}{(4m)^\alpha(4m)^\alpha} + \frac{2m}{(4)^\alpha(4m)^\alpha} + \frac{mn}{(6)^\alpha(6)^\alpha} \\
&- \frac{3m}{(6)^\alpha(6)^\alpha} + \frac{2mn}{(4)^\alpha(4)^\alpha} - \frac{3m}{(4)^\alpha(4)^\alpha} + \frac{4mn}{(4)^\alpha(4)^\alpha} - \frac{8m}{(4)^\alpha(6)^\alpha} \\
RR_\alpha(G) &= 2m^2(4m+2)^\alpha(4m+2)^\alpha - m(4m+2)^\alpha(4m+2)^\alpha + 4m^2(4m+2)^\alpha(4m)^\alpha \\
&+ 4m(4m+2)^\alpha(4)^\alpha + 2m(4m+2)^\alpha(6)^\alpha + 2m^2(4m)^\alpha(4m)^\alpha \\
&- m(4m)^\alpha(4m)^\alpha + 2m(4m)^\alpha(4)^\alpha + mm(6)^\alpha(6)^\alpha - 3m(6)^\alpha(6)^\alpha \\
&+ 2mn(4)^\alpha(4)^\alpha - 3m(4)^\alpha(4)^\alpha + 4mn(6)^\alpha(4)^\alpha - 8m(6)^\alpha(4)^\alpha \\
SDD(G) &= (4m^2 - 2m) + 4m^2\left(\frac{32m^2+16m+4}{16m^2+8m}\right) + 4m\left(\frac{16m^2+16m+20}{16m+8}\right) \\
&+ 2m\left(\frac{16m^2+16m+40}{24m+12}\right) + (6m^2 - 14m + 6mn + 2) + \frac{26mn - 52m}{3} \\
H(G) &= \frac{m^2}{2m+1} + \frac{4m^2}{4m+1} + m[\frac{2}{2m+3} + \frac{1}{2m+4} - \frac{1}{8m+4} + \frac{1}{2m+2} - \frac{3}{8} - \frac{4}{5}] \\
&+ mn[\frac{1}{12} + \frac{1}{4} + \frac{2}{5}] - \frac{1}{8} \\
I(G) &= m^4[\frac{24}{2m+1}] + m^3[\frac{12}{2m+1} + \frac{8}{4m+1}] + m^2[\frac{16}{2m+3} - \frac{2}{4m+1}] \\
&+ m[\frac{-4}{8m+4} - \frac{231}{5}] + mn[\frac{166}{3}] \\
A(G) &= \frac{(2m^2-m)(4m+2)^6}{(8m+2)^3} + \frac{4m^2(16m^2+8m)^3}{(8m)^3} \\
&+ \frac{4m(16m+6)^3}{(4m+4)^3} + \frac{2m(24m+12)^3}{(4m+6)^3} + \frac{(2m^2-m)(16m^2)^3}{(8m-2)^3} + \frac{2m(16m)^3}{(4m+2)^3} \\
&+ \frac{(mn-3m)(36)^3}{(10)^3} + \frac{(2mn-3m)(16)^3}{(6)^3} + \frac{(4mn-8m)(24)^3}{(8)^3}
\end{aligned}$$

Proof. Apply Formulas (22), (23), (24), (25), (26), (27), (28), (29), (30) and (31) to the edge partitions shown in the Table 2 to get the required results. □

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